Compensation of Timing Jitter-Induced Distortion of Sampled Waveforms

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Compensation of Timing Jitter-Induced Distortion of Sampled Waveforms

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Abstract—The presence of timing jitter between the trigger signal and the sampling strobe in an equivalent-time sampling oscilloscope causes distortion of the recorded waveform. Two methods exist to estimate the waveform from the jittered measurements. One method, called the median method, is based on the calculation of the point-by-point median of a large set of waveform measurements. It is shown here that this method is asymptotically biased if noise is present and if the waveform is nonmonotonic. Another method, called the pdf deconvolution method, is based on an estimation of the jitter probability density function and on a technique to deconvolve this density function from the average of all recorded waveforms. To estimate the jitter probability density function, it is assumed that the waveform has a part which can vary well be approximated by a ramp during a time span which is smaller than the standard deviation. It is shown that a significant asymptotic bias is introduced by the method when this assumption is violated. A novel approach is proposed, based on a parametric model of the jitter probability density function, which results in an asymptotic unbiased estimate of the jitter probability density function. The method is experimentally verified, and it is explained why this method is especially useful when one is interested in the Fourier spectrum of the recorded waveform.

I. INTRODUCTION

"TIME jitter" can cause significant systematic errors when waveforms are recorded with equivalent-time sampling oscilloscopes. The presence of time jitter means that every sample taken by the oscilloscope can only be situated on the time axis with a certain probability. This timing jitter causes systematic errors in the estimate of the measured waveform which are very hard to compensate because the jitter is a stochastic process very much dependent on the oscilloscope as well as the quality of the trigger signal. This means that the effect of the jitter will be different whenever we use another oscilloscope or another trigger signal! In the literature two methods are encountered that deal with waveform distortions due to time jitter: the so-called median method [1] and the so-called pdf deconvolution method [2]. The “median” method is based on the calculation of the point-by-point median of a large set of waveform measurements. Souders et al. [1] show that the estimate of the true waveform will be asymptotically biased when this waveform is nonmonotonic. We explain in this article that the presence of additive noise also introduces a bias.

These two sources of bias make the method useless if one wants an accurate estimation of a pulse waveform. The “pdf deconvolution” method is based on an estimation of the jitter probability density function (pdf) and on a technique to deconvolve this density function from the average of all recorded waveforms. The accuracy of this method is very much dependent on the quality of the estimate of the jitter probability density function [2]. Gans [2] uses a portion of the waveform that can be approximated by an ideal ramp and a measurement of the apparent additive noise pdf at one time instant to estimate the jitter pdf. Although useful in many practical cases, an asymptotic bias is present when no part of the waveform can be approximated very well by a ramp, as is the case for pulses when the time jitter standard deviation becomes too large relative to the pulse transition duration. We propose a novel method which results in an asymptotic unbiased estimate of the jitter pdf. The method extends the use of the pdf deconvolution method to cases where the “ideal ramp approximation” approach can no longer be used. The method is based on the identification of the parameters of a parametric model of the jitter probability density function. For the purpose of the parameter identification, an error function is minimized which is a function of the parameters, the mean of the measured waveforms, and the mean of the square of the measured waveforms.

In this paper, the method is derived mathematically, and it is shown what algorithm may be used to identify the parameters of a parametric model for the jitter pdf. It is explained how the effect of additive noise is effectively removed. A comparison is made between the median method and the novel method, based upon simulated data. Next the problems arising when applying the method of Gans [2] to estimate the jitter pdf are explained by illustrating the asymptotic bias of the method. Finally, the new method is applied to experimental data, illustrating its performance when the jitter standard deviation is large.

II. MATHEMATICAL EQUATIONS OF THE EXTENDED PDF DECONVOLUTION METHOD

For simplicity it is assumed that the sample rate with which the waveform is digitized is high enough to avoid aliasing and that care has been taken to avoid leakage problems. With these assumptions all continuous equations derived can easily be used for a digitized equivalent.

We assume that the jitter process can be characterized by a single pdf for all sampling instants. This pdf and
the undistorted waveform are the two unknowns. The two things that will be measured are the point-by-point average of all measured waveforms, and the point-by-point average of all measured waveforms squared (first squaring and then averaging). In fact, these two functions will be equivalent to using the mean and the variance of a set of recorded waveforms. At this point it should be clear to the reader that this variance is not a constant at all sampling instants when time jitter is present, but will be larger at those sampling instants where the slope of the waveform is larger.

Notations

\( x(t) \) Undistorted true signal.

\( a(t) \) Expectation of the measured signal (infinite number of averages).

\( s(t) \) Expectation of the square of the measured signal (infinite number of averages).

\( p(t) \) Time jitter probability density function.

\( X(\omega), A(\omega), S(\omega), P(\omega) \) Fourier transforms of \( x, a, s, \) and \( p. \)

In practice it will be possible to estimate \( a(t) \) and \( s(t) \) in an asymptotic unbiased manner by recording many waveforms and by calculating the mean \( a(t) \) and the mean of the waveforms squared \( s(t) \). It is possible to construct a set of equations describing the dependency of \( A \) and \( S \) on \( X \) and \( P. \)

In (1) the expectation of the measured waveform is calculated:

\[
a(t) = \int_{-\infty}^{\infty} x(t-\tau)p(\tau) \, d\tau. \tag{1}
\]

In this equation \( \tau \) is the stochastic variable indicating the amount of jitter at a certain moment of time, which shows up as a delay applied to the original signal. Equation (1) shows us that \( a(t) \) (this is the waveform we measure after having applied an infinite number of averages) is actually the convolution of the true signal \( x(t) \) and the probability density function of the jitter noise \( p(t) \). In the frequency domain this equation has the following form:

\[
A(\omega) = X(\omega)P(\omega). \tag{2}
\]

The measured averaged waveform spectrum equals the original signal spectrum \( X(\omega) \) multiplied by the characteristic function \( P(\omega) \) of the jitter noise. \( P(\omega) \) is the Fourier transform of the jitter noise probability density function. The idea developed by Gans [2] is to measure \( p(\tau) \) and to deconvolve it from the measured signal \( a(t). \) The main problem of course is how to measure \( p(\tau). \) Instead of trying to measure \( p(\tau) \) in a direct manner, which in some cases is hard to accomplish, a second equation independent of (2) can be derived. Equation (3) gives the relationship between \( s(t) \) and the two unknowns \( x(t) \) and \( p(\tau). \) Similar to the case of \( a(t) \) in (1), \( s(t) \) is the convolution of the square of \( x(t) \) and the pdf of the jitter noise \( p(\tau), \) that is,

\[
s(t) = \int_{-\infty}^{\infty} x^2(t-\tau)p(\tau) \, d\tau. \tag{3}
\]

In the frequency domain this equation becomes

\[
S(\omega) = (X^*X)(\omega)P(\omega) \tag{4}
\]

and (4) shows that \( S(\omega) \) equals the product of \( P(\omega) \) and the (frequency domain) convolution of \( P(\omega) \) with itself.

The final set of equations relating \( A \) and \( S, \) the measured waveforms, and the unknowns \( X \) and \( P \) are

\[
A(\omega) = X(\omega)P(\omega) \\
S(\omega) = (X^*X)(\omega)P(\omega). 
\]

According to the knowledge of the author, no algorithm is available in the literature to solve such a set of equations for a general \( P \) and \( X. \) When a parametric model for \( P(\omega) \) is proposed, however, it will become possible to estimate the parameters with good accuracy by minimizing an error function based upon (2) and (4). Before going into this, the following section covers the effects of additive noise.

III. Effects of Additive Noise

In experimental data additive noise will always be present. This noise, assumed here to be white, will cause minor problems for the algorithm. This can be shown as follows.

In practice, \( a(t) \) and \( s(t) \) will be estimated by taking the average of many realizations of the measured signal and the square of the measured signal. One realization of the measured signal can be described as \( x_m(t) = x(t-\tau) + n(t). \) In this equation \( \tau \) is a realization of the stochastic jitter variable \( \tau \) has a different value for each \( t, \) and \( n \) stands for a realization of the stochastic zero-mean additive noise variable. Now the relation between \( x(t), p(t) \) and \( a(t), s(t) \) can be recalculated. The expectation of the measured signal \( a(t) \) will equal the expectation of \( x(t-\tau) \) plus the expectation of \( n(t), \) because of the linearity of the expectation operator. Because \( n(t) \) is zero-mean additive noise, its expectation will equal zero, which means that \( a(t) \) will still be given by (1), and in the frequency domain, (2) stays valid. More care is needed, however, with \( s(t). \) In the following equations \( s(t) \) is calculated as a function of \( p(t), x(t) \) and \( \sigma_n, \) the standard deviation of the additive noise source \( n(t). \) The notation \( \langle \ldots \rangle \) denotes the expectation operator:

\[
s(t) = \langle x_m^2(t) \rangle = \langle x^2(t-\tau) + 2x(t-\tau) n(t) + n^2(t) \rangle \tag{5}
\]

\[
s(t) = \langle x^2(t-\tau) \rangle + 2 \langle x(t-\tau) \rangle \langle n(t) \rangle = \langle n^2(t) \rangle \tag{6}
\]

\[
s(t) = \int_{-\infty}^{\infty} x^2(t-\tau)p(\tau) \, d\tau + \sigma_n^2. \tag{7}
\]

When deriving (6) from (5) we used the fact that \( n \) and \( \tau \) are statistically independent. As can be seen from (7) this \( \sigma_n \) is squared and added as a constant to (3). In the frequency domain this constant will show up as a Dirac distribution \( \delta(\omega). \) The final set of frequency-domain equations is given by (8):

\[
A(\omega) = X(\omega)P(\omega) \\
S(\omega) = (X^*X)(\omega)P(\omega) + \sigma_n^2 2\pi \delta(\omega). \tag{8}
\]
IV. THE USE OF A PARAMETRIC MODEL FOR $P(\omega)$

According to the knowledge of the author, no algorithm is available in the literature to solve (8) for a general $X(\omega)$ and $P(\omega)$. It will be possible, however, to use a parametric model for $P(\omega)$ and to identify the parameters of this model. In the literature [3] several parametric models are used to model or at least sufficiently approximate most probability density functions and associated characteristic functions appearing in practice. For the simulations and the experiments mentioned in this paper the use of a normal distribution model for $p(\tau)$ appeared to be sufficient. In this section it is explained in a more general way, however, how the parameters can be identified when a model is used for $p(\tau)$. Whatever a parametric model is used, it is important to know that we can always assume that the expectation of $p(\tau)$ is equal to zero. It can theoretically be assumed that the expectation of $p(t)$ is different from zero, but this assumption will correspond to a pure delay applied on the digitized signal. One example of a parametric model for $p(\tau)$ that is currently under investigation corresponds with a so-called Edgeworth's form of the Type A series [3]. For $M$ parameters (called $\kappa_1$ until $\kappa_M$) the model for the corresponding characteristic function $P(\omega)$ is given by (9), when $s$ refers to the square root of $-1$. This model is equivalent to a truncation of the Taylor’s series of $\log(P(\omega))$, the so-called cumulant-generating function, and has several interesting characteristics [3]. Note that the characteristic function of a normal distribution model for $p(\tau)$ corresponds to (9) with $\kappa_1$ equal to the mean, $\kappa_2$ equal to the variation, and all other $\kappa_i$ equal to zero:

$$P(\omega, \kappa_1, \cdots, \kappa_M) = \exp \left( \sum_{i=1}^{M} \frac{\kappa_i}{j^i} (i\omega)^i \right)$$

(9)

It is now explained how the parameters of a parametric model for $P(\omega)$ can be identified when $A$ and $S$ are known. The parametric model for $P(\omega)$ will be noted as $P(\omega, \lambda_i)$, where $\lambda_i$ refers to $M$ parameters.

In order to explain the algorithm to identify the parameters $\lambda_i$, some mathematical notations will first be introduced. In practice digitized waveforms are used. The equivalent sampling time will be noted as $T_s$, and the number of points on the time axis used for the measurements will be noted $N$. The fundamental angular frequency of the discrete Fourier transform (DFT) will be noted as $\omega_{\text{base}}$ and is given by

$$\omega_{\text{base}} = \frac{2\pi}{NT_s}.$$  

(10)

To avoid confusion, a single-sided DFT is used: this means that only components will be considered with an index smaller than $N/2$. A subscript refers to the corresponding DFT component. $A_i^M$ and $S_i^M$ refer to the value of the $i$th component of the DFT of the measured average of the digitized waveform and the average of the waveform squared. If the number of averages is increased, $A_i^M$ and $S_i^M$ will asymptotically tend to $A(i\omega_{\text{base}})$ and $S(i\omega_{\text{base}})$. The notation $P(\lambda_i)$ will refer to $P(i\omega_{\text{base}}, \lambda_i)$. With these notations and conventions, (8) can be written in a discrete form. The result is given by (11):

$$A_i^M = X_i P_i(\lambda_i)$$

$$S_i^M = (X_i * X_{-i}) P_i(\lambda_i) + \sigma_i^2 \delta_{i0}.$$  

(11)

An asterisk denotes the single-sided DFT equivalent of a double-sided DFT circular convolution in the frequency domain, and $\delta_{i0}$ is a vector with the component with index 0 equal to 1 and all other components equal to 0. To calculate the convolution of two complex spectra, fast convolution techniques are used, performing inverse-DFT's (IDFT's), multiplications, and DFT's. The fast convolution algorithm calculates the convolution as shown in (12):

$$F * G = \text{DFT}^{-1}(\text{DFT}(F) \cdot \text{DFT}(G)).$$  

(12)

In a first step, $X_i$ is eliminated by substituting $X_i$ in the second equation of (11) by $A_i^M P_i^{-1}(\lambda_i)$. The result is written in (13):

$$S_i^M = ((A_i^M P_i^{-1}(\lambda_i)) (A_i^M P_i^{-1}(\lambda_i))) - P_i(\lambda_i) + \sigma_i^2 \delta_{0}.$$  

(13)

An error vector $e_i(\lambda_j)$ is then introduced based upon (13). It is defined by (14):

$$e_i(\lambda_j) = S_i^M - ((A_i^M P_i^{-1}(\lambda_j)) (A_i^M P_i^{-1}(\lambda_j)),$$

$$P_i(\lambda_j) - \sigma_i^2 \delta_{0}.$$  

(14)

To reduce the effects of noise, all components of $A_i^M$ that have an index larger than a certain $i_{\text{MAX}}$, corresponding to the highest frequency component that can be distinguished from the noise, will be made equal to zero. Since a large amount of oversampling is applied, this will always be possible. Based upon this error vector and an error function is introduced which is a function of the measured values $A_i^M$, $S_i^M$ and of the unknown parameters $\lambda_j$. This error function $r(\lambda_j)$ is defined by (15):

$$r(\lambda_j) = \sum_{i=1}^{2i_{\text{MAX}}} |e_i(\lambda_j)|^2.$$  

(15)

Note that this error function is not a function of the unknown $\sigma_i^2$ because the index $i = 0$ is eliminated from the summation. Also note that the largest component index in the summation is equal to $2i_{\text{MAX}}$. Considering components of $e_i(\lambda_j)$ with an index larger than $2i_{\text{MAX}}$ is theoretically possible but will only add a constant independent of $\lambda_j$ to the error function, which means that the final estimate will not be influenced. An estimate $\lambda_j^F$ for the true parameters $\lambda_j$ will be given by the value of $\lambda_j$ which minimizes the function $r(\lambda_j)$. By construction, $\lambda_j^F$ will be an asymptotic unbiased estimator for $\lambda_j^T$.

For the experiments and simulations mentioned here, a Levenberg–Marquardt algorithm [4] was used to minimize $r(\lambda_j)$. The method is based on an initial guess of the parameters (e.g., parameters that correspond to an absence of jitter)
an iterative process that converges to the final solution. The
algorithm is summarized in (16):
\[ \lambda^{N+1} = \lambda^N + (\text{Re}(J(\lambda^N) J^T(\lambda^N) + \Lambda))^{-1} \cdot \text{Re}(J(\lambda^N) e^i(\lambda^N)). \]

In this equation \( \lambda^N \) represents the \( N \)th approximation of \( \lambda^T \), \( J \)
is the Jacobian of the error vector \( e \), the superscript + means the transpose conjugate of a matrix, \( I \) is the identity matrix, \( \text{Re} \) refers to taking the real part of a complex matrix, and \( \Lambda \) is an algorithmic scalar parameter which systematically decreases when the algorithm is converging towards a solution. A typical stop criterion for the iteration is the convergence level reaching the computer machine precision.

Concerning the calculation of the Jacobian of the error vector, it may be useful to point out that it can easily be calculated despite the complex functional form of this error vector. The result of the calculation of the Jacobian is given in (17):
\[ \frac{\partial e}{\partial \lambda_j} = -((A^M P^{-1})(A^M P^{-1}) \frac{\partial P}{\partial \lambda_j}) + 2P((A^M P^{-1}) (A^M P^{-1}) \frac{\partial P}{\partial \lambda_j}). \]

As can be deduced from (13) an asymptotic unbiased estimate 
\( \sigma^2_{n|EST} \) for \( \sigma^2_n \) is given by (18):
\[ \sigma^2_{n|EST} = \sigma^2_0 - ((A^M P^{-1}(\lambda_j))(A^M P^{-1}(\lambda_j))h_0. \]

V. COMPARISON VERSUS MEDIAN METHOD

Before going into the comparison between the median method and the extended pdf deconvolution method, a short overview of the median method [1] will be given. The idea of the median method is the following. Assume a strictly monotonic waveform \( x(t) \) which is being sampled at a nominal time instant \( T_n \) relative to the trigger event. Due to the timing jitter the value of our sample will not equal \( x(T_n) \) but will equal \( x(T_n - \tau) \), with \( \tau \) being a stochastic variable. In a set of samples taken at nominal times \( T_n \), about half of the samples will be taken at time instants earlier than \( T_n \) and the other half at instants later than \( T_n \). Because of the strict monotonicity of \( x(t) \), this also means that the value of about half of the samples will be lower than \( x(T_n) \) and the other half higher. To have a good estimation of \( x(T_n) \) it is sufficient to calculate the median of all sample values. It is easy to prove that this estimator is asymptotically unbiased for monotonic waveforms when no additive noise is present. Unfortunately most waveforms that are measured are not monotonic. When this is the case all maxima and minima will be clipped [1], and no matter how many measurements are performed, this distortion can never be compensated. The inevitable presence of additive noise, presumed to be white with zero mean, will also cause a bias. This fact, not mentioned in [1], can be explained by the fact that the relation between the pdf of sampled values at a certain \( T_n \) when additive noise is present equals the ideal additive-noise-free pdf convolved with the pdf of the additive noise. Since the median of a distribution is by no means invariant with respect to convolutions if the distribution is nonsymmetric (even when the distribution to convolve with is a zero mean normal distribution), the presence of the additive noise will cause a bias in all points where the pdf of the sampled values is nonsymmetric.

In order to compare the performance of the median method with the newly developed method, software was written simulating the jitter process. The idea was to define an analytically described pulse. For convenience the impulse response of a third-order Butterworth low-pass filter was chosen. From this analytically defined waveform sampled versions could easily be calculated. This pulse corresponds to the theoretical \( x(t) \) in the previous derivation. Then several time-jittered versions were constructed by evaluating \( x(t) \), not at instants \( nT_n \), but at instants \( nT_n - \tau \), with \( \tau \) being a random number, different for each different version as well as for each sample. By applying the desired "numerical recipe" [5], many different probability density functions can be constructed out of the random numbers created by the computer (which usually have a uniform distribution). The average of all jittered versions was then calculated as well as the average of all jittered versions squared. These two functions are the estimates for \( \alpha(t) \) and \( \sigma(t) \) and are transformed into the frequency domain by an FFT, resulting in \( A(\omega) \) and \( S(\omega) \). The algorithm of Section IV was then applied with a normal distribution model for \( p(\tau) \). This resulted in an estimate for \( P(\omega) \). Using the method as described by Gans [2] to deconvolve this \( P(\omega) \) from \( A(\omega) \) resulted in an estimate for \( X(\omega) \). The median method estimator was also applied to the jittered waveforms, so finally the quality of both methods could be compared. For the simulation the following parameters were chosen:

\[ x(t) = \begin{cases} 
 1 - 1/2 \sin \left( \frac{\sqrt{3} t}{2} \right) - \cos \left( \frac{\sqrt{3} t}{2} \right), & \text{for } t > 0 \\
 0, & \text{for } t < 0.
\end{cases} \]

applied jitter noise: Gaussian with \( \sigma_{jitter} = 0.75 \); 
applied additive noise: Gaussian with \( \sigma_n = 0.03 \) V; 
T: 0.3 s (sampling time); 
number of samples: 256; 
start time: -10 s (stop time = 66.5 s); 
Nyquist frequency = 5 Hz; 
number of simulated waveforms used for averaging, 3000; 
number of simulated waveforms used for median estimate, 3000.

The results of the simulation are shown in Figs. 1 through 4. A discussion of these results is noted in what follows. 

Fig. 1 and Fig. 2 show the results of both methods in the time domain. The median method underestimates the maximum voltage of the true waveform by about 12%, while the error for this maximum voltage is only about 0.25% with the extended pdf deconvolution method. The large error for the median method is due to the previously mentioned clipping effect [1]. As can be seen from Fig. 1 and Fig. 2, the pdf deconvolution method can accurately estimate the
maximum voltage, but it introduces a ringing effect which is not present with the median method. This ringing is typical for the deconvolution process and can also be noted in the results of Gans [2]. The bias introduced by the median method due to the presence of additive noise can be seen when looking at the voltage values for time instants close to 0 s. A bias is present, although the waveform is perfectly monotonic at this time instant. This is in agreement with the theory as explained in the beginning of this section.

Fig. 3 and Fig. 4 show the results of both methods in the frequency domain. The result of the median method is significantly biased at all frequencies. This can easily be explained by the clipping effect. The result of the pdf deconvolution method is very accurate for frequencies smaller than 0.2 Hz, but the results for frequencies higher than 0.3 Hz are very inaccurate. This 0.3 Hz corresponds to the frequency where the spectral components of \( A(\omega) \) have amplitude values which become comparable to the noise floor. Such a frequency will always be present for the "pdf deconvolution" method. It is a limit for the method, which causes the estimates of the spectral components of the true signal to be accurate only for frequency components smaller than a certain limit frequency. This fact causes the time-domain ringing effect mentioned above. Note that the limit frequency will increase when more averaging is applied because this will result in a lower value for the noise floor. Such a limit frequency does not exist for the median method. This method will be able to estimate the values of spectral components with frequencies that are considerably higher than the pdf deconvolution method frequency limit. Note, however, that the estimates will be biased.

To conclude it can be said that the decision of which method to choose depends upon the application. When ringing can be allowed or an accurate estimate is needed of signal spectra, the extended pdf deconvolution method gives very good results. If the waveform is monotonic and there is not too much additive noise present, the median method is preferable.

VI. ASYMPTOTIC BIAS OF THE CLASSICAL WAY TO ESTIMATE THE JITTER PDF

In this section the problems mentioned in the introduction, concerning the method to determine the jitter pdf as mentioned in [2], are illustrated.

Suppose we are sampling the waveform described by (19), with the same jitter pdf as in the previous section, but with no additive noise present. In this case the method as described in [2] would use the measurement of the slope of the averaged waveform at a certain time instant, which will be called \( \tan \theta \), together with the standard deviation of the vertical noise at this instant, called \( \sigma_M \), to estimate the jitter standard deviation \( \sigma_{\text{jitter}} \). The relation between the two measured quantities and the estimate of the jitter standard deviation, denoted by \( \bar{\sigma}_{\text{jitter}} \), is then given by (20):

\[
\bar{\sigma}_{\text{jitter}} = \frac{\sigma_M}{\tan \theta}.
\]  

It is easy to show [2] that this method gives an asymptotic unbiased estimate for \( \sigma_{\text{jitter}} \) if applied to an ideal ramp. If applied to pulse-like signals, however, with a jitter standard deviation which is significant relative to the pulse transition duration, an asymptotic bias will result on the estimation of \( \sigma_{\text{jitter}} \). This will cause an error on the estimation of the spectrum of the undistorted signal. We illustrate this on the waveform as described by (19).
TABLE I

<table>
<thead>
<tr>
<th>time instant (seconds)</th>
<th>averaged value (volts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.1345</td>
</tr>
<tr>
<td>0.8</td>
<td>0.1821</td>
</tr>
<tr>
<td>1.1</td>
<td>0.2403</td>
</tr>
</tbody>
</table>

Fig. 5. Measured spectra of averaged SRD pulse with different attenuation in trigger path. Y scale in dBm, X scale in GHz. Solid: 0 dB of attenuation in trigger path; dashed: 10 dB of attenuation in trigger path; dot-dashed: 15 dB of attenuation in trigger path.

We choose the time instant to measure \( \sigma_M \) and \( \tan \theta \) equal to 0.8 s. This instant is chosen such that the value of the averaged waveform is about half the maximum value achieved. At this instant the pulse-like signal can best be approximated by a ramp. Next the conditions of the previous section are used and 3000 jittered waveforms are simulated. The averaged waveform is calculated for time instants equal to 0.5, 0.8 and 1.1 s. The result is given in Table I.

From these three values shown in Table I, the value of \( \tan \theta \) is estimated to be equal to 0.1763 V/s. This value is calculated by using linear regression. Next the standard deviation \( \sigma_M \) is calculated for a time instant equal to 0.8 s. The result is 0.1450 V. Using (20) this results in a value for \( \bar{\sigma}_{\text{jitter}} \) equal to 0.8224 s. The exact value of \( \bar{\sigma}_{\text{jitter}} \) is equal to 0.75 s. We can conclude that there will be an asymptotic bias of about 10% on the estimate of \( \bar{\sigma}_{\text{jitter}} \), due to the fact that the waveform is not an ideal ramp. When this biased value of \( \bar{\sigma}_{\text{jitter}} \) is used in the deconvolution algorithm the error in the estimation of the spectral component with a frequency of 0.2 Hz would be about 0.8 dB. This asymptotic bias is not present when the method is used as described in Section IV. It is important to note, however, that the bias mentioned in this section becomes negligibly small when the waveform can be approximated very well by a ramp over an interval large compared to \( \bar{\sigma}_{\text{jitter}} \). In such a case both methods have the same accuracy, but the method described by Gans [2] is much simpler to implement.

VII. EXPERIMENTAL RESULTS

Finally an experimental setup was made to check the new method in practice. To accomplish this, a pulse generated by a step recovery diode (SRD) was measured by an HP 54120 sampling oscilloscope, and the extended pdf deconvolution method was applied. To control the amount of jitter, a controllable attenuator was introduced in the trigger signal path.

By attenuating the trigger signal the amount of jitter could artificially be enlarged. Then it became possible to compare the reconstructed waveforms with less and the ones with more jitter. This way it can be shown that the reconstructed waveforms are insensitive to jitter and that the method works well. The results of the experiment are shown in Fig. 5 through Fig. 7. The SRD was excited by a 97.65625-MHz sine wave; the oscilloscope took 1024 samples with a sampling time equal to 10 ps. For calculating the point-by-point average of the waveforms and the waveforms squared, 1000, 2000, and 4000 averages were used corresponding to 0 dB, 10 dB, and 15 dB of attenuation in the trigger path, respectively.

Fig. 5 shows the spectra of the three averaged waveforms, corresponding to an attenuation of 0 dB, 10 dB, and 15 dB, respectively. The low-pass filtering effect of the jitter can clearly be distinguished. At 7 GHz, for example, the third spectrum (15 dB of attenuation in trigger path) has an attenuation of 30 dB relative to the first spectrum (0 dB of attenuation in trigger path). Fig. 6 shows the same spectra when the extended pdf deconvolution method has been applied. The parametric model that was used for the pdf corresponds to a normal distribution. Now the three spectra are practically coincidental. The fact that the third spectrum no longer corresponds to the other two above 7 GHz is due to the fact that at 7 GHz the spectrum of the averaged third waveform goes down into the noise floor. This means that the compensating filter will be amplifying noise for this spectrum once above 7 GHz. Fig. 7 shows the differences between the reconstructed spectra with 10 dB and 15 dB of attenuation in the trigger path relative to the reconstructed spectrum with 0 dB in the trigger path. This shows that the
three spectra correspond very well for those frequencies where
the averaged waveform spectrum does not reach the noise
floor (correspondence within about 200 mDB). Two parasitic
effects can be distinguished. First an offset of about 200 mDB
is noticed in Fig. 7. This can be explained by the fact that
the different experiments took place over a very long period
(a whole night) and that there might have been a small gain
drift of the oscilloscope during the experiment. A second
effect is the difference being substantially larger for the first 5
frequency components. This is probably caused by a leakage
effect of the periodic excitation component of the SRD which
is dominantly present in the measurement (cf. the peak near dc
in Fig. 5 and Fig. 6). Although the frequency of this component
was chosen carefully to avoid leakage, an inevitable small
error in the oscilloscope’s timebase accuracy will still cause a
small amount of leakage of this component, having a power
about 11 dB higher than the surrounding components. Because
the phase of this leaky component is different relative to the
sampling window whenever another attenuation level is used
in the trigger path, its effect will be different from experiment
to experiment, and this will show up as it does in Fig. 7.

The estimated values for the jitter standard deviation for the
three cases are 6.4 ps, 15.8 ps, and 57.6 ps, respectively.

VIII. CONCLUSION

A new method has been described for the determination of
the characteristic function of the jitter pdf. This method can be
used to extend the pdf deconvolution method as described by
Gans [2] to cases where the waveform cannot be approximated
by a ramp at any time instant, which is the case for pulse-like
signals when the standard deviation of the jitter is no longer
small compared with the pulse transition duration. The method
uses the average and the average of the square of a large set
of recorded waveforms. It has been shown that the method is
especially useful when a correct reconstruction of the spectrum
is required.

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