Individual Characterization of Broadband Sampling Oscilloscopes with a 'Nose-to-Nose' Calibration Procedure

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Individual Characterization of Broadband Sampling Oscilloscopes with a Nose-to-Nose Calibration Procedure

Jan Verspecht and Ken Rush

Abstract — A method is proposed to find the individual impulse response of broadband sampling oscilloscopes (HP54124T, 50 GHz bandwidth). The method is based on the fact that, when the oscilloscope is sampling a dc voltage, pulses are launched from the sampler towards the input connector. These pulses contain information on the scope's characteristic and can be measured by a second oscilloscope. This type of measurement is called a "nose-to-nose" calibration. Applying deconvolution techniques to the result of this measurement, the characteristic of the two scopes can be found, assuming that the two scopes are identical. To avoid relying on this assumption, three oscilloscopes can be used. The nose-to-nose is then applied three times, with a different pair of scopes connected together each time. The individual characteristics of the three sampling oscilloscopes are then calculated. Both SPICE simulations and close correspondence between swept-sine measurements and this method indicate that it is probably the most accurate method available at this moment to calibrate broadband sampling oscilloscopes. Several measurement uncertainties and practical problems are identified. The phase contribution of the sampling aperture can never be determined by this method, but there is strong evidence that the effect is negligible. Practical measurement problems are related to linearity of the samplers, additive noise, time-base jitter, and time-base drift.

I. INTRODUCTION

Many problems arise when one wants to calibrate broadband sampling oscilloscopes. With the present technology, sampling oscilloscopes are available with bandwidths exceeding 50 GHz, for example, the Hewlett-Packard HP54124T, with a specified bandwidth of 50 GHz, and a typical bandwidth of about 70 GHz. Before the development of the "nose-to-nose" calibration method [1], mainly two methods were mentioned in the literature to measure the linear characteristics of broadband sampling oscilloscopes. The first method is based on a detailed modeling of the oscilloscope sampling circuitry [2]. To be useful, an accurate knowledge is needed of the values of parasitic elements. For the latest generation of sampling oscilloscopes, with extremely small sampler dimensions, the values of parasitic elements are very hard to determine. This means that the modeling approach can hardly be used. A second method [3], [4] is based on the availability of a "pulse standard," this is a pulse generator with an accurately known pulse shape. The idea is to connect the pulse standard generator to the oscilloscope and to digitize the measured waveform. Using deconvolution techniques, the oscilloscope's linear characteristics can be calculated from the knowledge of the "standard pulse" shape and the digitized waveform. The main problem with this approach is the availability of a pulse standard. In [3], the pulse standard itself is characterized by means of electro-optic sampling, in [4] by the use of a superconducting sampling oscilloscope. In both cases, the assumption was made that the sampling circuitry with which the pulse standard was measured was perfect, and could be used as a kind of "oscilloscope standard." The problem is how to characterize your oscilloscope standard, which actually returns us to the initial problem. A possible way out of this was discovered in 1990 by the designers of the Hewlett-Packard HP54124T sampling oscilloscope [1]. The method is called the nose-to-nose calibration procedure. It is based on the fact that a sampling oscilloscope sampling dc acts as a pulse generator. This pulse, named "kick-out" and launched from the sampler head towards the input connector, contains information on the oscilloscope's linear characteristic. By digitizing this pulse with a second oscilloscope and using deconvolution techniques, the linear characteristic of the oscilloscope can be extracted from the waveform of the pulse finally appearing on the screen of the pulse-receiving oscilloscope. This work will cover simplified theoretical models, some comments concerning SPICE simulations on more complex models, and experimental verification of the nose-to-nose calibration procedure by comparing the result with "swept-sine" measurements. A short description of measurement uncertainties and practical problems is included.

II. THEORETICAL DESCRIPTION

A. Mathematical Notations

F(t): An uppercase letter to designate a function denotes a real function of the time variable t.

f(ω): A lowercase letter denotes the Fourier transform of the time function indicated by the same letter in uppercase, e.g., f(ω) is the Fourier transform of F(t).

f ∗ g: Denotes the convolution of the functions f and g.

f* : Denotes the complex conjugate of f.


\begin{align}
\begin{array}{c}
b_2(\omega) = s_{21}(\omega)a_1(\omega) + s_{12}(\omega)a_2(\omega) \\
b_2(\omega) = s_{21}(\omega)a_1(\omega). 
\end{array}
\end{align}

In these equations, \( a_1, a_2, b_1, \) and \( b_2 \) are equal to the classical voltage waves defined as follows:

\begin{align}
\begin{array}{c}
a_1(\omega) = \frac{v_1(\omega) + Z_e z_1(\omega)}{2} \\
a_2(\omega) = \frac{v_2(\omega) + Z_e z_2(\omega)}{2} \\
b_1(\omega) = \frac{v_1(\omega) - Z_e z_1(\omega)}{2} \\
b_2(\omega) = \frac{v_2(\omega) - Z_e z_2(\omega)}{2}. 
\end{array}
\end{align}

Next, we write the equations describing the effect of the time-varying conductance \( G(t) \):

\begin{align}
I_c(t) &= V_2(t)G(t) \\
I_2(t) &= -V_2(t) \left( \frac{1}{Z_e} + G(t) \right). 
\end{align}

Next we define a function \( P(t) \) by the following equation:

\begin{align}
P(t) &= \frac{Z_e G(t)}{2 + Z_e G(t)}. 
\end{align}

Using (4) and (6) in the time domain and combining this with (8) and (9) results in the following equation:

\begin{align}
A_2(t) = -P(t)B_2(t). 
\end{align}

In the frequency domain (10) becomes

\begin{align}
a_2(\omega) = -p(\omega) * b_2(\omega). 
\end{align}

Using (4), (6), (9), and (10), we are able to transform (7) into the following relationship:

\begin{align}
I_c(t) &= \frac{2}{Z_e} P(t)B_2(t). 
\end{align}

Calculation of “kick-out” pulse shape: We will now calculate the shape of the so-called kick-out pulse. This is the pulse that is launched from the sampler head towards the input connector when a dc voltage is being sampled. This pulse shape will be described by \( K(t) \), defined as a voltage wave appearing at the input connector. To calculate \( K(t) \), we first make \( A_1(t) \) equal to the constant value of 1 V. We will then use (1)-(11) to calculate \( B_1(t) \), which, by definition, will equal \( K(t) \). In the frequency domain, we can write

\begin{align}
a_1(\omega) = 2\pi \delta(\omega). 
\end{align}

Substituting this in (2) we find

\begin{align}
b_2(\omega) = 2\pi s_{21}(0) \delta(\omega). 
\end{align}

Looking at the physical structure (see Fig. 1), we can see that \( s_{21}(0) = 1 \). Substituting (14) into (11) and the result of this into (1), we find \( k(\omega) \):

\begin{align}
k(\omega) = 2\pi s_{11}(0) \delta(\omega) - 2\pi s_{12}(\omega)p(\omega). 
\end{align}
Taking a look at Fig. 1, we can see that \( s_{12}(0) \) equals 0. The final result then becomes

\[
k(\omega) = -2\pi s_{12}(\omega)p(\omega).
\]

(16)

**Calculation of impulse response:** To calculate the impulse response of our sampler, we will make \( A_1(t) \) equal to a Dirac-impulse. To know what will be the sampled value as a function of the sampling instant, we will then drive the samplers at a time instant \( \tau \) (by substituting \( P(t-\tau) \) for \( P(t) \)), and we will calculate the integral value of \( I_c(t,\tau) \) for \( t \) going from minus infinity to plus infinity. This integral value will correspond to the charge stored on the hold capacitors, which is proportional to the value displayed on the screen of the oscilloscope at sampling instant \( \tau \). Making \( A_1(t) \) equal to a Dirac-impulse gives us the following relation in the frequency domain:

\[
a_{1}(\omega) = 1.
\]

(17)

Substituting this into (2) and transforming into the time domain results in:

\[
B_2(t) = S_{21}(t).
\]

(18)

Substituting this in (12) with the introduction of the variable \( \tau \), we find

\[
I_c(t,\tau) = \frac{2}{Z_c} P(t-\tau)S_{21}(t).
\]

(19)

If we denote the sampled value that will be displayed on the scope’s display at time instant \( \tau \) by \( V(\tau) \), we can write

\[
V(\tau) = L \int_{-\infty}^{\infty} I_c(t,\tau)dt = \frac{2L}{Z_c} \int_{-\infty}^{\infty} P(t-\tau)S_{21}(t)dt
\]

(20)

with \( L \) being an arbitrary integration constant. Finally, this equation can be transformed into the frequency domain. The result is

\[
v(\omega) = N \cdot p^+(\omega)s_{21}(\omega)
\]

(21)

with \( N \) now being an arbitrary constant.

**Nose-to-nose experiment:** When we perform a nose-to-nose measurement, the inputs of two oscilloscopes are connected together. Next, a dc voltage is put on the hold capacitors of one of the oscilloscopes; this oscilloscope will be the pulse creator. This oscilloscope will sample, and the kick-out pulse, as described by (16), will appear at its input connector. This pulse will then be sampled by the second oscilloscope, whose impulse response is given by (21). In the following, it will be assumed that \( s_{11}^A s_{11}^B \), with superscript \( A \) and \( B \) referring to the two oscilloscopes, is much smaller than 1 and can be neglected. By using three oscilloscopes, named \( A, B, \) and \( C, \) we will be able to find the individual characteristic of each oscilloscope. To achieve this, three measurements will be performed, connecting each time a different couple of oscilloscopes together and by doing a nose-to-nose experiment. The results of these measurements, called \( m_{AB}(\omega), m_{AC}(\omega), \) and \( m_{BC}(\omega), \) with the subscript referring to the oscilloscopes connected together, are calculated by using (16) and (21). The results are given by the following equations:

\[
m_{AB}(\omega) = M_{AB}s_{21}^A(\omega)s_{12}^B(\omega)p^{+*}(\omega)p^B(\omega)
\]

(22)

\[
m_{AC}(\omega) = M_{AC}s_{21}^A(\omega)s_{12}^C(\omega)p^{+*}(\omega)p^C(\omega)
\]

(23)

\[
m_{BC}(\omega) = M_{BC}s_{21}^B(\omega)s_{12}^C(\omega)p^{+*}(\omega)p^C(\omega)
\]

(24)

with \( M \) referring to arbitrary constants.

From these three equations, we can find an estimate for the impulse response of, for example, oscilloscope \( A \). We now define \( v_{ext}^A(\omega) \) as follows:

\[
v_{ext}^A(\omega) = \sqrt{m_{AB}(\omega)m_{AC}(\omega)\over m_{BC}(\omega)}.
\]

(25)

In (25) the square root is taken of a complex function. To avoid phase ambiguity, phase unwrapping is necessary before the square root is taken. Using (22)–(24), and the fact that the \( s \)-parameter network in Fig. 2 is reciprocal, we can find the relation between the Fourier transform of the impulse response of oscilloscope \( A \), as given by (21) and \( v_{ext}^A(\omega) \)

\[
v_{ext}^A(\omega) = M^A(\omega)\sqrt{p^B(\omega)\over p^{+*}(\omega)}
\]

(26)

with \( M \) a constant that can be determined by a dc measurement.

This can also be written as:

\[
v_{ext}^A(\omega) = M^A(\omega)e^{j\phi(p^B(\omega))}.
\]

(27)

In practice there is strong evidence to state that \( p^B(t) \) is practically symmetric. This is based on the fact that the strobe pulse is formed by combining a very steep step-like signal (generated by a step recovery diode) with its delayed inverse. This is done by letting the step-like signal reflect on a short. The pulse \( P(t) \), as defined by (9), will be correlated with the top of this strobe pulse (the part of the strobe pulse exceeding the threshold created by the reverse bias voltage of the sampling diodes). As such, \( P(t) \) will have a symmetric form, such that \( e^{j\phi(p^B(\omega))} \) may very well be approximated by 1. Looking at (27) shows that \( v_{ext}^A(\omega) \) will be a good estimate for the Fourier transform of the impulse response of oscilloscope \( A \). Calculating the inverse Fourier transform of \( v_{ext}^A(\omega) \) results in an estimation of the impulse response. An example of such a measurement is given in Fig. 3.

### III. Early SPICE Modeling Results

We started doing SPICE [5] modeling looking for nonsymmetric aperture effects, realizing that any nonsymmetry effects in the sampling process will be masked by the nose-to-nose measurement methods. What we found was that any nonsymmetry effect in the sampling aperture was minimal compared to the random noise or other nonlinearity effects in the timebase or voltage measurement process. However, the nonlinearity of the sampling diode capacitance (which fundamentally violates the assumption of network reciprocity) causes different charge injection profiles into the output nodes versus the input node. This gives rise to a fundamental difference between the sampling kick-out and the impulse response,
Fig. 3. Measured impulse response of a 54124A test set.

Fig. 4. Measured step response of a 54124A test set.

with the kick-out waveform showing more overshoot. The response estimate of the nose-to-nose measurement could reasonably be expected to lie between the two. Based on the SPICE modeling work, we believe that the overshoot in the measured sampler step response (Fig. 4) could be overstated by as much as 2% for the HP 54124A in the high-bandwidth mode. Further work needs to be done to determine the correct methods of separating kick-out waveforms from step response waveforms.

IV. PRACTICAL MEASUREMENT SETUP

The experimental setup for actually performing a nose-to-nose measurement is illustrated in Fig. 5. Only the pulse receiving oscilloscope, named scope B, is connected to a controller with an IEEE488-interface bus. One input connector of the scope A (kick-out creating oscilloscope) is connected to one input of scope B. Channel 1 of scope B is connected to the trigger input of scope A. Scope B is put in “time-domain reflectometry”-mode (TDR-mode); this means that scope B will generate steps at the connector of channel 1, and will take samples relative to the instant of the steps. Scope A is put in “HISTOGRAMMING”-mode; this means that it will take a sample (i.e., create a kick-out pulse) as soon as it is triggered by the steps created by scope B. In theory, scope A should be sampling a dc voltage. In practice, this is achieved by applying an offset voltage on the hold capacitors of scope A. This can be done by use of the built-in “OFFSET” function. The kick-out pulse created when sampling a dc voltage of 100 mV without any offset voltage, is equivalent to the kick-out pulse created when sampling a zero voltage with an offset voltage on the hold capacitors of −100 mV. A parasitic effect, not yet mentioned, occurs with the practical setup because not only the kick-out appears at the input connector, but also a portion of the strobe pulse which is coupled onto the input channel. To remove this strobe pulse, scope B is made to sample two kick-out pulses, one created with −100 mV offset on scope A and another one with 100 mV offset on scope A. The strobe pulse will be present in both measurements with the same sign, while the kick-out pulse will be present once with a positive and once with a negative sign. By subtracting the positive-offset measurement from the negative-offset measurement, the effect of the strobe pulse is successfully removed.

V. SWEEP-SINE MEASUREMENT

Results of nose-to-nose and swept-sine measurements were published by Rush, Draving, and Kerley [1]. Subsequently, many more measurements have been made, and a procedure has been documented [11] to allow others to make the same nose-to-nose measurements. A discussion of these techniques follows. Figs. 6–9 show four different setups for making these measurements. Fig. 6 shows a setup for measuring the power delivered by a synthesized sweeper into a 50 Ω load for frequencies below 26.5 GHz. Details of this measurement can be found in the user’s manuals for the various measurement instruments. This setup assumes that swept-sine measurements are to be made on the HP 54124A Four-Channel Test Set which has 2.4 mm connectors. The 6 dB pad minimizes the effects of standing waves on the SMA cable. We are assuming that high-quality GORE cables are being used which are in good repair, i.e., having no dirty connectors or damage at connector ends, and no dents or bruises. The connectors must be torqued to the specified amount recommended by the vendor of the connectors. The measured power at several frequencies, for which calibration data are available from HP, should be made and then the corrections applied for gain and return loss. Measurements should be made down to the lower limit available from the synthesizer to get a good value for a dc reference.
Fig. 6. Setup to measure synthesizer output power (below 26.5 GHz).

Fig. 7. Setup used to measure swept-sine response of the HP 54124A Digitizing Oscilloscope (below 26.5 GHz).

Fig. 8. Setup to measure synthesizer output power (above 26.5 GHz).

Fig. 9. Setup used to measure swept-sine response of the HP 54124A Digitizing Oscilloscope (above 26.5 GHz).

Fig. 10. Comparison of swept-sine response and frequency amplitude response derived from the nose-to-noise measurements.

Fig. 7 shows a setup for measuring the scope’s response below 26.5 GHz. Note that the output from the sweeper, the cable, and all adapters and attenuators are the identical ones used in the leveling measurements. The connectors should not be loosened between the two measurements. The trigger for the scope is obtained from the synthesizer’s 10 MHz reference output.

The RF amplifier is used to provide large signals to the scope’s trigger input. Note that if the amplifier is used, the output will be saturated and quite distorted. This does not harm the amplifier, although its output looks more like a square wave than a sine wave. This is good in that the trigger system can trigger on a high slew-rate signal while not having to absorb high power. Frequencies must be selected that are multiples of the 10 MHz reference. Actually, if the synthesizer uses a frequency multiplier circuit on its output, as is the case with the HP 8340A, the input of the multiplier must be a multiple of the 10 MHz reference. The multiplier in use is dependent on the output frequency band. This is specified in the user’s manual for the synthesizer.

Fig. 8 shows the leveling calibration for frequencies above 26.5 GHz. The band was broken into two sections: 26.5 GHz–33 GHz and 33 GHz–50 GHz, and a different waveguide mm-wave source was used in each section. These measurements were made before continuous 10 MHz to 50 GHz sweepers were available in 2.4-mm coax. The measurements would be much easier with such an instrument. At these frequencies great care must be taken to insure that connectors are properly cleaned and torqued to insure repeatable results. Descriptions of how to set up leveled waveguide signals are available in the manuals of the various instruments.

Fig. 9 shows the scope measurements above 26.5 GHz. Again, the exact same hardware as used in the leveling measurements must be used here. A trigger, in this case, is obtained by sending the auxiliary output signal (which is the frequency that goes into the frequency multiplier) through the HP 54118A 18 GHz trigger and then into the scope’s trigger input. One must be sure to adjust the trigger level and hold-off control for maximum jitter.

If great care is taken in both the swept-sine measurements and the nose-to-noise measurements, then comparison results like those in Fig. 10 can be achieved. Fig. 10 shows comparisons between the calculated amplitude response achieved by the nose-to-noise measurements and the swept-sine measurements for both the maximum bandwidth mode and the reduced bandwidth mode available from the HP 54124A. For each bandwidth mode, the smoothest curve corresponds to the amplitude response derived from the nose-to-noise measurements.
VI. MEASUREMENT UNCERTAINTIES AND PRACTICAL PROBLEMS

A. Introduction

In the aforementioned, three approaches are applied to support the quality of the nose-to-nose calibration method: a theoretical approach based upon a simplified model, simulations, and a cross-check with a swept-sine method. From this we can conclude that the nose-to-nose calibration method should be considered as a serious candidate for establishing an international standard for picosecond pulse measurements. One remaining question, however, is the question of the measurement uncertainty. As yet, research concerning this topic is on going. Some more comments on this will be given in the following.

B. Modeling Errors

A first question concerns the validity of the simplified model of the theoretical approach, illustrated in Fig. 2.

In the model, the switching diodes are modeled as a linear time-variant conductance \( G(t) \). This assumption will be valid as long as the sampler’s behavior is linear. This can easily be verified in practice by increasing the offset voltage and by looking at the measurement results. If the sampler behaves linearly, all measured kick-out pulses will differ only by a constant factor. At a certain voltage, however, a small nonlinear behavior can be noticed. With an offset voltage of 100 mV, the error on the amplitude response due to the nonlinear behavior is smaller than 250 mdB for a frequency of 30 GHz.

Because of the assumption that the RC constant of the sampler circuitry (100 picoseconds) is much larger than the switching duration (10 picoseconds), the hold capacitors are modeled as connections to ground. More sophisticated simulations were performed with conditions corresponding to RC constants of 25 picoseconds and switching durations of about 30 picoseconds, with several shapes of conductance versus time (both rectangular and Gaussian). These simulations show that the general assumption of kick-out being equal to impulse response is valid in practice even in these conditions (only very small, negligible errors are found). From this can be concluded that the modeling error due the assumption of the RC constant being infinite (by replacing the hold capacitor by a short) is negligible.

For the moment, it is unclear how the assumption of \( s_{22} \) being equal to zero affects the measurement uncertainty. From the physical structure of the input channel, we may conclude that the amplitude of \( s_{22} \) will approximately equal the amplitude of \( s_{11} \), which can be measured by connecting the input connector to a vector network analyzer. Such a measurement showed a value of \( s_{11} \) being smaller than −25 dB from dc to 20 GHz.

An important issue is the assumption of \( P(t) \) being symmetric, such that its Fourier transform has no phase contribution. Although there is evidence that this may be assumed without making too much of an error, a bound can be derived for this error under less stringent assumptions. With the only assumption that \( G(t) \) is different from zero only during a certain interval with length \( T_{ap} \), it can be proven [6] that the deviation of the phase of \( p(\omega) \) from linearity, denoted \( \varphi(\omega) \), satisfies the following inequality:

\[
|\varphi(\omega)| \leq \arctan \left( \frac{\omega T_{ap} \sin(\omega T_{ap})}{\cos(\omega T_{ap})} \right), \quad \text{for} \ 2\omega T_{ap} \leq \pi.
\]

(28)

A graph of this uncertainty can be found in Fig. 11, assuming a \( T_{ap} \) of 10 picoseconds and using the frequency \( f \) instead of the angular frequency \( \omega \). Table I shows some of the phase values.

It is important to note, however, that the range of the phase dispersion will be much less than the worst case theoretical limit shown.

C. Measurement Errors

Noise: One of the major problems is the relatively high-noise energy relative to the energy of the kick-out pulse. With the following settings, the noise floor of the measured frequency characteristic is only about 20 dB lower than the main low-frequency components:

- time range: 2048 picoseconds
- number of points: 1024
- number of averaging: 64
- scope A offset voltage: 100 mV.

These settings were chosen to avoid aliasing (the Nyquist frequency being equal to 250 GHz), and to assure that leakage is not present. To verify that there is no leakage present, the step response was calculated by integrating the impulse response. It was then noticed that the step response no longer changes after about 600 picoseconds.

In order to lower the noise floor, a large number of averages is needed. Looking at (25) reveals that the noise floor will go down following a 5 log (N) law, with \( N \) being the number
of averages. This means that in order to lower the noise floor by 15 dB, 1000 times the number of averages are needed. Although not impossible, this means that the measurement time can be very long if a small uncertainty due to noise is wanted.

**Jitter noise:** In the measurement setup, a small amount of timing jitter will be present. This jitter will have a low-pass filtering effect [7]. Using the method described in [8], the jitter standard deviation was estimated to be about 1.5 picoseconds. Without using compensation for this error [7] and [8]), the error would have the approximate magnitude given in Table II. This jitter compensation was not used for the results given in Fig. 10.

**Timebase drift:** When a lot of averaging is needed, a measurement can take a very long time (several hours). During this time, care has to be taken to avoid environmental temperature changes. These temperature drifts can cause a small drift of the timebase relative to the trigger instant. A typical value is about 2 picoseconds per degree Celsius. When regular averaging is applied, the effect of the drift can be compared to the effect of the timing jitter, meaning that it will appear as if an additional low-pass filtering had taken place. This distortion can, however, be removed if, instead of a regular averaging technique, a so-called logarithmic spectral-averaging technique is applied. With this technique, the average is calculated of the complex logarithm of the discrete Fourier transform (DFT) of each measured pulse. By doing this, the timebase drift will not affect the estimate of the measured pulse spectrum. This can be explained by modeling the s-th pulse measurement, called $M_s(t)$ as follows:

$$M_s(t) = X(t - 	au_s) + N_s(t)$$  \hspace{1cm} (29)

with $\tau_s$ being the stochastic time drift and $N_s(t)$ being the additive noise of the s-th measurement. Calculating the Fourier transform of (29) results in

$$m_s(\omega) = x(\omega)e^{-j\omega\tau_s} + n_s(\omega).$$  \hspace{1cm} (30)

The logarithmic spectral average will be called $A_{\log}(\omega)$, and it is defined as follows:

$$A_{\log}(\omega) = \frac{1}{K} \sum_{s=1}^{K} \ln(x(\omega))e^{-j\omega\tau_s} + n_s(\omega)).$$  \hspace{1cm} (31)

This can be written as follows:

$$A_{\log}(\omega) = \ln(x(\omega)) - \frac{1}{K} \sum_{s=1}^{K} \ln(1 + \frac{n_s(\omega)}{x(\omega)})$$  \hspace{1cm} (32)

with $n_s(\omega)$ having the same stochastic properties as $n_s(\omega)$.

Next, we calculate the expectation of $A_{\log}(\omega)$. This results in the following expression:

$$< A_{\log}(\omega) > = \ln(x(\omega)) + j\omega < \tau_s > + \text{Bias}(\sigma_n)$$  \hspace{1cm} (33)

where $\text{Bias}(\sigma_n)$ denotes a bias which is a function of the inverse of the signal-to-noise ratio, called $\sigma_n$, and the probability density function of the additive noise. As was theoretically proven in [9], the analytical expression of $\text{Bias}(\sigma_n)$ for noise with a Gaussian probability density function is the following:

$$\text{Bias}(\sigma_n) = -\frac{1}{2} E_i \left( \frac{-1}{2\sigma_n^2} \right)$$  \hspace{1cm} (34)

where $E_i$ denotes the “exponential-integral” function [10]. A plot of this function is given in Fig. 12. As can be seen on the graph, the bias will have a value of less than 5 mB as soon as the signal-to-noise ratio is higher than 10 dB, which is the case in a practical measurement. Looking at (34), we see that the expectation of the logarithmic spectral averager will actually equal $\ln(x(\omega))$, plus an imaginary part which is a linear function of $\omega$. For our purpose this linear imaginary part will not be of any importance since it corresponds to a delay of the measured pulse, which will not disturb our measurement.

To conclude this topic, we can say that the measurements must either be performed under tight temperature control or that the logarithmic spectral-averaging technique should be used.

**VII. CONCLUSION**

A method, called nose-to-nose calibration procedure, has been described which enables the individual characterization of broad-band equivalent time-sampling oscilloscopes. In this article, the method was illustrated on oscilloscopes of the Hewlett-Packard 5412x series. It is important to notice, however, that the theoretical approach works for all sampling systems with the same topological structure, whatever the physics of the switching process (electro-optics or semiconductivity) are. The method has the advantage that the calibration bandwidth will always be equal to the bandwidth of the sampling system itself.

Although promising, it is clear that, in order to become a standard, a lot of work still has to be done concerning
the quantitative determination of the measurement uncertainties. At this moment we believe to have identified the most important sources of error.

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REFERENCES


Jan Verspecht, for a photograph and biography, see page 215 of this TRANSACTIONS.

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His current interests include genetic probability mathematics and neural networks as applied to the stock market.