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Review

Scale relativity theory and integrative systems biology: 2 Macroscopic quantum-type mechanics

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Abstract

In these two companion papers, we provide an overview and a brief history of the multiple roots, current developments and recent advances of integrative systems biology and identify multiscale integration as its grand challenge. Then we introduce the fundamental principles and the successive steps that have been followed in the construction of the scale relativity theory, which aims at describing the effects of a non-differentiable and fractal (i.e., explicitly scale dependent) geometry of space–time. The first paper of this series was devoted, in this new framework, to the construction from first principles of scale laws of increasing complexity, and to the discussion of some tentative applications of these laws to biological systems. In this second review and perspective paper, we describe the effects induced by the internal fractal structures of trajectories on motion in standard space. Their main consequence is the transformation of classical dynamics into a generalized, quantum-like self-organized dynamics. A Schrödinger-type equation is derived as an integral of the geodesic equation in a fractal space. We then indicate how gauge fields can be constructed from a geometric re-interpretation of gauge transformations as scale transformations in fractal space–time. Finally, we introduce a new tentative development of the theory, in which quantum laws would hold also in scale space, introducing complexergy as a measure of organizational complexity. Initial possible applications of this extended framework to the processes of morphogenesis and the emergence of prokaryotic and eukaryotic cellular structures are discussed. Having founded elements of the evolutionary, developmental, biochemical and cellular theories on the first principles of scale relativity theory, we introduce proposals for the construction of an integrative theory of life and for the design and implementation of novel macroscopic quantum-type experiments and devices, and discuss their potential applications for the analysis, engineering and management of physical and biological systems and properties, and the consequences for the organization of transdisciplinary research and the scientific curriculum in the context of the SYSTEMOSCOPE Consortium research and development agenda.

Keywords: Scale relativity; Systems biology; Scale covariance; Macroscopic quantum mechanics; Self-organization

Contents

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1. Introduction

Caminante, no hay camino, se hace camino al andar—Antonio Machado

In the first paper, we provided a first indication that scale invariant laws and generalized scale laws with variable fractal dimensions have the potential to found elements of the evolutionary, developmental and cellular biology theories on the common first principles of scale relativity theory. In this second paper we explore the effects of fractal structures on the laws of dynamics in standard space, then in scale space, before

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discussing the possible consequences of this extended framework for multiscale integration in systems biology and the development of novel experiments and devices with macroscopic quantum-type behaviours.

2. New macroscopic quantum-type mechanics

2.1. Motivation

Let us now consider an essential part of the theory of scale relativity, namely, the description of the effects in standard space–time that are induced by the internal fractal structures of its geodesics. The companion paper introduced pure scale laws describing the dependence on scale of fractal paths at a given point of space–time. The next step consists of considering a displacement of such a structured point, i.e., the consequences on motion of the non-differentiability. As we shall see, these consequences are radical since they amount to a transformation of Newton's equation of dynamics into a generalized Schrödinger equation.

Note that, in the perspective of potential applications to biological systems, we consider here only the non-relativistic case (i.e., velocities small with respect to the velocity of light c). One can show (Nottale, 1993) that this case corresponds to a fractal space, while time keeps its regular behaviour.

As a first step, we shall mainly consider only the simplest case of fractal internal structures, namely, the self-similar ones that are characterized by a constant fractal dimension, and more precisely fractal dimension $D_F = 2$ that plays a critical role in the theory (Nottale, 1996a). As we have seen in the companion paper, this behaviour can be derived from a simple, scale-inertial differential equation of first order. We shall see that the laws of mechanics constructed from such internal structures of the geodesics of a fractal space become a quantum-type mechanics. Therefore the various generalizations of internal scale laws that have been considered in the companion paper naturally lead to generalized quantum laws, as we shall briefly see in Section 2.3.7.

Actually, the discovery that typical quantum mechanical paths (those that contribute mainly to the path integral) are non-differentiable and of fractal dimension 2 is due to Feynman (Feynman and Hibbs, 1965), even though the word "fractal" was coined by Mandelbrot only in 1975. But the various properties of quantum paths described by Feynman in his approach (which is not a return to determinism, since his paths are in infinite number) correspond very closely to the later definition of fractals (Abbott and Wise, 1981; Ord, 1983; Nottale and Schneider, 1984). Now, Feynman derives the fractal and non-differentiable properties of quantum paths from quantum mechanics and its sets of axioms, while the scale relativity approach attempts to do the reverse, namely, found quantum mechanics on the non-differentiable and fractal geometry of space–time.

2.2. Method

The method is as follows. We start from a generic description of the displacements in a non-differentiable and continuous space, which is fractal as a consequence, following the fundamental founding theorem of the theory (Nottale, 1993, 1996a; Cresson, 2001, 2003). As we shall see, the paths in a fractal space are characterized by three minimal properties: fractality, infinite number and time irreversibility.

These three conditions are mathematically expressed at the level of the elementary displacements, then their effect on a physical quantity is described in terms of a "quantum-covariant" derivative. This means that, since the dynamical effects of a space geometry are internal (instead of being external as in the case of an externally added force or field), they are included in the differential calculus itself (Einstein, 1916). But in addition to the general (motion) relativity case, in the scale relativity case we have to deal not only with the effects of the geometry, but also with those of the non-differentiability (which does not mean that we cannot define differential elements, but that their ratios, i.e., the derivatives, are sometimes undefined).

Finally, the principle of relativity–equivalence–covariance allows one to write the equation of geodesics as a free form motion equation, which expresses the acceleration $\widehat{d}^2 X/dt^2 = 0$ in terms of the new covariant derivative $\widehat{d}$ (see its construction in what follows). This means that one writes that, locally, there is rectilinear uniform motion, so that all the final complexity comes from the change of reference system itself. The final

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step amounts to make changes of variables (without any change of the number of degrees of freedom) which transform the classical type of physico-mathematical tool into a quantum-type tool.

2.2.1. Fractality of the paths

Strictly, the non-differentiability of the coordinates means that the standard velocity

$$V(t) = \frac{\mathrm{d}X}{\mathrm{d}t} = \lim_{\mathrm{d}t \to 0} \frac{X(t+\mathrm{d}t) - X(t)}{\mathrm{d}t} \tag{1}$$

is undefined. Namely, when $\mathrm{d}t$ tends to zero, either the ratio $\mathrm{d}X/\mathrm{d}t$ tends to infinity, or it fluctuates without reaching any limit.

However, as recalled above, continuity and non-differentiability imply an explicit dependence on scale (and even a divergence) of the various physical quantities. As a consequence, the coordinate $X(t)$ and the velocity, $V(t)$ are themselves re-defined as explicitly scale-dependent functions $X(t, \mathrm{d}t)$ and $V(t, \mathrm{d}t)$. We can therefore use again all the arguments developed in the companion paper, and conclude that, in the simplest case, we expect it to be solution of a first order scale differential equation, i.e.,

$$V(t, \mathrm{d}t) = v(t) + w(t, \mathrm{d}t) = v(t)\left\{1 + \eta(t)\left(\frac{T}{\mathrm{d}t}\right)^{1-1/D_{\mathrm{F}}}\right\}. \tag{2}$$

Here the position on the curve is now located by using the time t itself as parameter s, while the resolution is a time resolution. The scale T must be introduced for dimensional reasons, and appears as a constant of integration. Its presence manifests once again the fact that only scale ratios do have a physical meaning, not the scales themselves.

This result means that the velocity is now the sum of two independent terms of different orders of differentiation, since their ratio v/w is, from the standard viewpoint, infinitesimal (see Fig. 1). The v component is what we have called the "classical part" (or differentiable part) of the velocity (Célérier and Nottale, 2004), and w is its "fractal part" (or non-differentiable part). The new component w is an explicitly scale-dependent fractal fluctuation (see Fig. 4 of companion paper) which is described in terms of a dimensionless normalized stochastic variable $\eta(t)$ such that $\langle\eta\rangle = 0$ and $\langle\eta^2\rangle = 1$. As we shall see later on, the final result is totally independent of the probability distribution of this variable.

Eq. (2) multiplied by $\mathrm{d}t$ gives the elementary displacement, $\mathrm{d}X$, of the system as a sum of two infinitesimal terms of different orders

$$\mathrm{d}X = \mathrm{d}x + \mathrm{d}\xi, \tag{3}$$

which are such that

$$\mathrm{d}x = v\,\mathrm{d}t, \tag{4}$$

$$\mathrm{d}\xi = \eta(2\mathscr{D})^{1-1/D_{\mathrm{F}}}\,\mathrm{d}t^{1/D_{\mathrm{F}}}, \tag{5}$$

where the parameter $\mathscr{D}$ is a reformulation of the previous scale T of Eq. (2).

Only the critical case of fractal dimension $D_{\mathrm{F}} = 2$ will be now considered (see Nottale, 1995, 1996a, b and Section 2.3.7 for generalization to a different fractal dimension). The fluctuation becomes

$$\mathrm{d}\xi = \eta\sqrt{2\mathscr{D}}\,\mathrm{d}t^{1/2}. \tag{6}$$

The fundamental parameter $\mathscr{D}$ is bound to play a very important role in what follows. It can be considered to be defined by the above relation, namely,

$$\mathscr{D} = \frac{1}{2}\frac{\langle \mathrm{d}\xi^2\rangle}{\mathrm{d}t}. \tag{7}$$

It looks like a coefficient of diffusion, but here its meaning is of geometric essence, namely, it manifests the intrinsic diffusive property of a fractal space, but no external agent or particle is the cause of this "diffusion". This coefficient intervenes in the determination of the fundamental transition from scale dependence (fractality) to scale independence (see Fig. 1). But, as we shall see, it may also be identified, modulo

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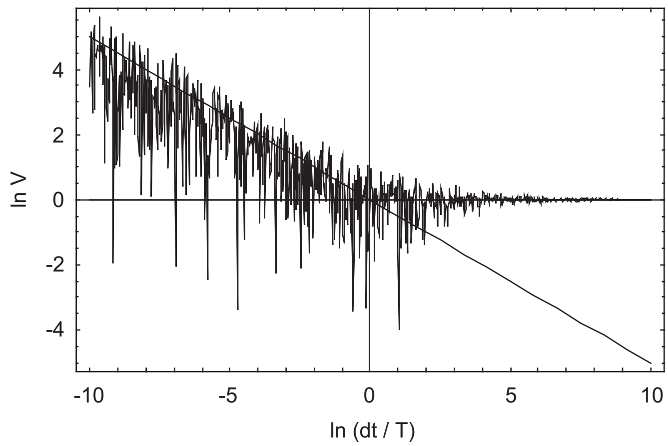

Fig. 1. Dependence on the time-scale, $\ln(dt/T)$, of the logarithm of the velocity $\ln(V/V_0)$ on a fractal geodesic, including a classical (differentiable) part, which is dominant at large scale (it tends to a scale-independent velocity toward the right in the figure), and a fractal (non-differentiable) fluctuating part, which is dominant at small scales (to the left). The fluctuation has been taken here to be Gaussian. The fluctuating fractal part $d\xi$ is of order $dt^{1/2}$ for fractal dimension 2, so that the velocity diverges toward small scales as $dt^{-1/2}$, which expresses the non-differentiability of the fractal coordinate.

fundamental constants, to a generalization of the Compton scale, $\hbar/mc$, that is the fundamental wavelength which has phenomenologically appeared in quantum mechanics (without having, up to now, been theoretically understood from first principles).

2.2.2. Infinite number of geodesics

One of the main geometric consequences of the non-differentiability is that there is an infinity of fractal geodesics relating to any couple of points of a fractal space (Nottale, 1993; Cresson, 2001). This can be easily understood already at the level of fractal surfaces, which can be described in terms of a fractal distribution of conic points of positive and negative infinite curvature (see Nottale, 1993, Sections 3.6 and 3.10). As a consequence, we are led to replace the velocity $V(t, dt)$ on a particular geodesic by the fractal velocity field $V[x(t, dt), t, dt] = v[x(t), t] + w[x(t, dt), t, dt]$ of the whole infinite ensemble of geodesics. This representation is similar to that of fluid mechanics (Landau and Lifchitz, 1959), in which the motion of a fluid is described in terms of its velocity field $v(x(t), t)$, its density $\rho(x(t), t)$ and possibly its pressure. We shall indeed recover the fundamental equations of fluid mechanics (Euler and continuity equations), but written in terms of a density of probability (as defined by the set of geodesics) instead of a density of matter, and with an additional term of quantum pressure which occurs as a manifestation of the underlying fractal geometry (see below).

2.2.3. Discrete symmetry breaking from irreversibility

A last fundamental consequence of the non-differentiability is the breaking of a discrete symmetry, namely, of the reflection invariance on the differential element of time (it is said to be discrete since it is not a continuous symmetry, such as e.g., translation or rotation, but a discontinuous one such as a mirrored symmetry). It implies a two-valuedness of velocity which can be subsequently shown to be the origin of the fundamental use of complex numbers in quantum mechanics (Célérier and Nottale, 2004). This use determines a large part of the particularities of quantum mechanics with respect to classical mechanics.

The derivative with respect to the time t of a differentiable function f can be written twofold

$$\frac{df}{dt} = \lim_{dt \to 0} \frac{f(t + dt) - f(t)}{dt} = \lim_{dt \to 0} \frac{f(t) - f(t - dt)}{dt}. \quad (8)$$

The two definitions are equivalent in the differentiable case. In the non-differentiable situation, both definitions fail, since the limits are no longer defined. In the new framework of scale relativity, the physics is related to the behaviour of the function during the zoom operation on the time resolution δt, identified with the differential element dt. The non-differentiable function $f(t)$ is replaced by an explicitly scale-dependent

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fractal function $f(t, \mathrm{d}t)$, which therefore is a function of two variables, t (in space–time) and $\mathrm{d}t$ (in scale space). The two functions f'_+ and f'_- are therefore defined as explicit functions of the two variables t and $\mathrm{d}t$

$$f'_+(t, \mathrm{d}t) = \frac{f(t + \mathrm{d}t, \mathrm{d}t) - f(t, \mathrm{d}t)}{\mathrm{d}t}, \tag{9}$$

$$f'_-(t, \mathrm{d}t) = \frac{f(t, \mathrm{d}t) - f(t - \mathrm{d}t, \mathrm{d}t)}{\mathrm{d}t}. \tag{10}$$

Here we have assumed that $\mathrm{d}t > 0$. By taking $\mathrm{d}t$ algebraic, these two functions would correspond, respectively, to the positive and negative parts of a same unique function. One passes from one definition to the other by the transformation $\mathrm{d}t \leftrightarrow -\mathrm{d}t$ (differential time reflection invariance), which actually was an implicit discrete symmetry of differentiable physics. When applied to fractal space coordinates $x(t, \mathrm{d}t)$, these definitions yield, in the non-differentiable domain, two velocity fields instead of one, that are fractal functions of the resolution, $V_+[x(t), t, \mathrm{d}t]$ and $V_-[x(t), t, \mathrm{d}t]$. Each of these fractal velocity field can in turn be decomposed in terms of a classical part and a fractal part, namely, $V_+[x(t, \mathrm{d}t), t, \mathrm{d}t] = v_+[x(t), t] + w_+[x(t, \mathrm{d}t), t, \mathrm{d}t]$ and $V_-[x(t, \mathrm{d}t), t, \mathrm{d}t] = v_-[x(t), t] + w_-[x(t, \mathrm{d}t), t, \mathrm{d}t]$.

The important fact appearing here is that *there is no a priori reason for the two classical parts to be the same*. In several works Ord (Ord and Deakin, 1996; Ord and Galtieri, 2002) also insists on the importance of introducing entwined paths for understanding quantum mechanics (but without giving a cause for this fundamental two-valuedness), while Jumarie (2006) supports the scale-relativistic view that the use of complex-valued variables appears as a direct consequence of the irreversibility of time.

A simple and natural way to account for this doubling consists of using complex numbers $a + \mathrm{i}b$ and the complex product, according to which $(a + \mathrm{i}b)(c + \mathrm{i}d) = (ac - bd) + \mathrm{i}(ad + bc)$. This is the origin of the complex nature of the wave function of quantum mechanics. Actually, the choice of complex numbers to represent the two-valuedness of the velocity can be proven to be a simplifying and covariant choice (Célérier and Nottale, 2004; Nottale, 2008), in the sense of the principle of covariance, according to which the form of the equations of physics should be conserved under all transformations of coordinates. Indeed, the choice of the complex product allows one to suppress what would be additional infinite terms in the final equations of motion.

Another consequence of the combination of the two velocity fields into a single complex velocity field is that, in terms of this physical tool, one recovers a global reversibility of physical laws, as we are going to see from the derivation of the Schrödinger equation.

2.2.4. *Covariant total derivative operator*

We are now led to describe the elementary displacements for both processes, $\mathrm{d}X_\pm$, as the sum of a classical part, $\mathrm{d}x_\pm = v_\pm \mathrm{d}t$, and of a fractal fluctuation $\mathrm{d}\xi_\pm$, i.e.,

$$\mathrm{d}X_+(t) = v_+\mathrm{d}t + \mathrm{d}\xi_+(t),$$
$$\mathrm{d}X_-(t) = v_-\mathrm{d}t + \mathrm{d}\xi_-(t), \tag{11}$$

and similar relations for the other variables. One passes from one process to the other by the transformation $\mathrm{d}t \leftrightarrow -\mathrm{d}t$. More generally we define two classical derivatives, $\mathrm{d}_+/\mathrm{d}t$ and $\mathrm{d}_-/\mathrm{d}t$, such that

$$\frac{\mathrm{d}_+}{\mathrm{d}t}x(t) = v_+, \quad \frac{\mathrm{d}_-}{\mathrm{d}t}x(t) = v_-. \tag{12}$$

These expressions are also valid for three space variables by considering that x and v represent vectors.

The two derivatives can now be combined to construct a complex derivative operator that allows recovering local differential time reversibility in terms of the new complex process (Nottale, 1993). We define it as

$$\frac{\widehat{\mathrm{d}}}{\mathrm{d}t} = \frac{1}{2}\left(\frac{\mathrm{d}_+}{\mathrm{d}t} + \frac{\mathrm{d}_-}{\mathrm{d}t}\right) - \frac{\mathrm{i}}{2}\left(\frac{\mathrm{d}_+}{\mathrm{d}t} - \frac{\mathrm{d}_-}{\mathrm{d}t}\right). \tag{13}$$

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This choice is motivated by the need to recover, at the classical limit (where the two velocities are equal), the classical real velocity as real part of this complex velocity and a vanishing imaginary part. This is the main tool of the theory.

Applying this operator to the classical part of the position vector yields a complex velocity

$$\mathscr{V} = \frac{\widehat{d}}{dt} x(t) = \frac{v_+ + v_-}{2} - i\frac{v_+ - v_-}{2}. \tag{14}$$

We call V the real part of this complex velocity and U its imaginary part, i.e. $\mathscr{V} = V - iU$, with $V = v_+ + v_-/2$ and $U = v_+ - v_-/2$.

After having defined the covariant derivative, we now need to find its expression. This will be achieved by explicitly calculating its effect on a given physical quantity.

For this purpose, let us first calculate the derivative of a scalar function f. Since the fractal dimension is 2, we need to go to second order of expansion (this is reminiscent of Einstein's argument about Brownian motion). For one variable it reads

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial X}\frac{dX}{dt} + \frac{1}{2}\frac{\partial^2 f}{\partial X^2}\frac{dX^2}{dt}. \tag{15}$$

Once again the generalization of this writing to three dimensions is straightforward (Nottale, 1993).

Let us now take the stochastic mean of this expression (i.e., we take the mean of the stochastic variable η which appears in the definition of the fractal fluctuation $d\xi$). By definition, since $dX = dx + d\xi$ and $\langle d\xi \rangle = 0$, we have $\langle dX \rangle = dx$, so that the second term is reduced (in three dimensions) to $v.\nabla f$. (Recall that this expression denotes the scalar product of the velocity $v = (v_x, v_y, v_z)$ by the gradient $\nabla f = (\partial f/\partial x, \partial f/\partial y, \partial f/\partial z)$ of the function f, i.e., in decompacted form, $v.\nabla f = v_x\, \partial f/\partial x + v_y\, \partial f/\partial y + v_z\, \partial f/\partial z$).

Now concerning the term dX^2/dt, it is infinitesimal and therefore not taken into account in the standard differentiable case. But in the non-differentiable case considered here, the mean squared fluctuation is non-vanishing and of order dt, namely, $\langle d\xi^2 \rangle = 2\mathscr{D}\, dt$, so that the last term of Eq. (15) amounts in three dimensions to a Laplacian (defined as $\Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$). We obtain, respectively, for the (+) and (−) processes,

$$\frac{d_\pm f}{dt} = \left(\frac{\partial}{\partial t} + v_\pm.\nabla \pm \mathscr{D}\Delta\right) f. \tag{16}$$

The last step consists of recombining the two derivatives into the complex covariant derivative. Substituting Eqs. (16) into Eq. (13), we finally obtain the expression for the covariant time derivative operator (Nottale, 1993)

$$\frac{\widehat{d}}{dt} = \frac{\partial}{\partial t} + \mathscr{V}.\nabla - i\mathscr{D}\Delta. \tag{17}$$

This is one of the main tools of the theory of scale relativity. Indeed, the passage from standard classical (i.e., almost everywhere differentiable) mechanics to the new non-differentiable theory can now be implemented by replacing the standard time derivative d/dt by the new complex operator $\widehat{d}/dt$ (Nottale, 1993).

Note that in this replacement, one should remain aware of the fact that this derivative operator is a linear combination of first and second order derivatives, in particular when dealing with the Leibniz rule about derivatives of a product and of composed functions. Now it is possible to build more efficient, fully covariant, tools (Pissondes, 1999). This can be made by introducing a velocity operator $\widehat{\mathscr{V}} = \mathscr{V} - i\mathscr{D}\nabla$ (Nottale, 2004), in terms of which the first order Leibniz rule still applies, since the covariant derivative now reads

$$\frac{\widehat{d}}{dt} = \frac{\partial}{\partial t} + \widehat{\mathscr{V}}.\nabla. \tag{18}$$

In other words, this means that $\widehat{d}/dt$ plays the role of a "covariant derivative operator", namely, we are able, by using it, to write the fundamental equations of physics under the same form they had in the differentiable case.

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2.3. Covariant mechanics in scale relativity

Let us now summarize the main steps by which one may generalize the standard classical mechanics using this covariance. We consider below only the classical parts of the variables, which are differentiable and independent of resolutions. But the same final equation can be proved to be valid also for the full velocity field including its non-differentiable part, after redefinition of the wave functions in terms of fractal functions, see Nottale (2008).

We define a Lagrange function $\mathscr{L}(x, \mathscr{V}, t)$ that keeps the usual form but now in terms of the complex velocity, then a complex action $\mathscr{S}$ which is still defined as

$$\mathscr{S} = \int_{t_1}^{t_2} \mathscr{L}(x, \mathscr{V}, t)\, \mathrm{d}t. \tag{19}$$

One finds that generalized Euler–Lagrange equations that keep their standard form (see companion paper) in terms of the new complex variables can be derived from this action (Célérier and Nottale, 2004), namely

$$\frac{\widehat{\mathrm{d}}}{\mathrm{d}t}\frac{\partial \mathscr{L}}{\partial \mathscr{V}} - \frac{\partial \mathscr{L}}{\partial x} = 0. \tag{20}$$

Since we now consider only the classical parts of the variables (while the effects on them of the fractal parts are included in the covariant derivative) the basic symmetries of classical physics hold. From the homogeneity of standard space, one defines a generalized complex momentum given by the same form as in standard mechanics, namely,

$$\mathscr{P} = \frac{\partial \mathscr{L}}{\partial \mathscr{V}}. \tag{21}$$

If we now consider the action as a function of the upper limit of integration in Eq. (19), the variation of the action from a trajectory to another nearby trajectory yields a generalization of another well-known relation of standard mechanics,

$$\mathscr{P} = \nabla \mathscr{S}. \tag{22}$$

2.3.1. Geodesic form of the motion equations

Let us now consider the special case of Newtonian mechanics, in the general case when the structuring external scalar field is described by a potential energy Φ. The Lagrange function of a closed system, $L = \frac{1}{2}mv^2 - \Phi$, is generalized as $\mathscr{L}(x, \mathscr{V}, t) = \frac{1}{2}m\mathscr{V}^2 - \Phi$. The Euler–Lagrange equations then keep the form of Newton's fundamental equation of dynamics $F = m\,\mathrm{d}v/\mathrm{d}t$, namely, for a force that derives from a potential,

$$-\nabla\Phi = m\frac{\widehat{\mathrm{d}}}{\mathrm{d}t}\mathscr{V}, \tag{23}$$

which is now written in terms of complex variables and complex operators.

In the case when there is no external field ($\Phi = 0$), the covariance is explicit, since Eq. (23) takes the free form of the equation of inertial motion, i.e., of a geodesic equation,

$$\frac{\widehat{\mathrm{d}}}{\mathrm{d}t}\mathscr{V} = 0. \tag{24}$$

This is analog to Einstein's general relativity, where the equivalence principle of gravitation and inertia leads to a strong covariance principle, expressed by the fact that one can always find a coordinate system in which the metric is locally Minkowskian. This means that, in this coordinate system, the covariant equation of motion of a free particle is that of inertial motion $\mathrm{D}u_\mu/\mathrm{d}s = 0$ in terms of the general-relativistic covariant derivative D, four-vector u_μ and proper time differential ds. The expansion of the covariant derivative subsequently transforms this free-motion equation in a local geodesic equation in a gravitational field.

The covariance induced by scale effects leads to an analogous transformation of the equation of motions, which, as we show below, become after integration the Schrödinger equation, which we can therefore consider as the integral of a geodesic equation. Note that one also obtains in the motion-relativistic case the Klein–Gordon equation (Nottale, 1994a, 1996a) and the Dirac equation on spinors (Célérier and Nottale, 2004), since spinors arise as a consequence of a new two-valuedness from the breaking of the symmetry $dx \leftrightarrow -dx$ (see also Célérier and Nottale, 2006 about the Pauli equation).

In the Newtonian case the complex momentum $\mathcal{P}$ reads

$$\mathcal{P} = m\mathcal{V}, \tag{25}$$

so that, from Eq. (22), the complex velocity $\mathcal{V}$ appears as a gradient, namely the gradient of the complex action

$$\mathcal{V} = \nabla\mathcal{S}/m. \tag{26}$$

2.3.2. Emergence of the quantum tools

Up to now the various concepts and variables used were of a classical type (space, geodesics, velocity fields), even when they were generalized to the fractal, explicitly scale-dependent case.

We shall now make essential changes of variable, which transform this classical-like tool (that will finally reveal not to be classical) into quantum tools (but without any hidden parameter or degree of freedom). We now introduce a complex wave function ψ which is nothing but another expression for the complex action $\mathcal{S}$ by making the transformation

$$\psi = e^{i\mathcal{S}/S_0}. \tag{27}$$

The factor S_0 has the dimension of an action (i.e., an angular momentum) and must be introduced because $\mathcal{S}$ is dimensioned while the phase should be dimensionless. When this formalism is applied to microphysics, S_0 is nothing but the fundamental constant $\hbar$ of standard quantum mechanics. As a consequence, since

$$\mathcal{S} = -iS_0 \ln\psi, \tag{28}$$

one finds that the function ψ is related to the complex velocity appearing in Eq. (26) as follows:

$$\mathcal{V} = -i\frac{S_0}{m}\nabla(\ln\psi). \tag{29}$$

Since we have $\mathcal{P} = -iS_0\nabla\ln\psi = -iS_0(\nabla\psi)/\psi$, we obtain the equality $\mathcal{P}\psi = -i\hbar\nabla\psi$ (Nottale, 1993) in the standard quantum mechanical case $S_0 = \hbar$, which establishes a correspondence between the classical momentum p, which is the real part of the complex momentum in the classical limit, and the operator $-i\hbar\nabla$. Therefore the correspondence principle is no longer an independent axiom as it is in standard quantum mechanics.

2.3.3. Schrödinger form of the motion equation

We have now at our disposal all the mathematical tools needed to write the fundamental equation of dynamics (23) in terms of the new quantity ψ. It takes the form

$$iS_0\frac{\widehat{d}}{dt}(\nabla\ln\psi) = \nabla\Phi. \tag{30}$$

This equation can be integrated in a general way under the form of a Schrödinger equation.

Such an equation could be integrated provided its left-hand side be a gradient. But one should be aware that $\widehat{d}$ and ∇ do not commute. However, as we shall now see, $\widehat{d}(\nabla\ln\psi)/dt$ is nevertheless a gradient.

Replacing $\widehat{d}/dt$ by its expression, given by Eq. (17), yields

$$\nabla\Phi = iS_0\left(\frac{\partial}{\partial t} + \mathcal{V}.\nabla - i\mathcal{D}\Delta\right)(\nabla\ln\psi), \tag{31}$$

and replacing once again $\mathcal{V}$ by its expression in Eq. (29), we obtain

$$\nabla\Phi = iS_0\left[\frac{\partial}{\partial t}\nabla\ln\psi - i\left\{\frac{S_0}{m}(\nabla\ln\psi.\nabla)(\nabla\ln\psi) + \mathcal{D}\Delta(\nabla\ln\psi)\right\}\right]. \tag{32}$$

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This expression may be simplified thanks to the remarkable identity (see Nottale, 1993 and its proof in Appendix A),

$$\nabla\left(\frac{\Delta\psi}{\psi}\right) = 2(\nabla \ln \psi.\nabla)(\nabla \ln \psi) + \Delta(\nabla \ln \psi). \tag{33}$$

We recognize, in the right-hand side of the identity (33), the two terms of Eq. (32), which were, respectively, in factor of S_0/m and $\mathscr{D}$. Therefore, in order to simplify the right-hand side of Eq. (32), the arbitrary parameter S_0 in the definition of the wave function can be taken to be

$$S_0 = 2m\mathscr{D}. \tag{34}$$

One can prove that the final result is actually independent of this choice, see Nottale (2008).

Note that this relation is more general than the standard quantum mechanical one, in which S_0 is restricted to the only value $S_0 = \hbar$. The function ψ in Eq. (27) is therefore now defined as

$$\psi = e^{i\mathscr{S}/2m\mathscr{D}}, \tag{35}$$

so that the fundamental equation of dynamics now reads

$$\nabla\Phi = 2im\mathscr{D}\left[\frac{\partial}{\partial t}\nabla \ln \psi - i\{2\mathscr{D}(\nabla \ln \psi.\nabla)(\nabla \ln \psi) + \mathscr{D}\Delta(\nabla \ln \psi)\}\right]. \tag{36}$$

Now using the above remarkable identity and the fact that $\partial/\partial t$ and ∇ commute, it becomes

$$-\frac{\nabla\Phi}{m} = -2\mathscr{D}\nabla\left\{i\frac{\partial}{\partial t}\ln \psi + \mathscr{D}\frac{\Delta\psi}{\psi}\right\}. \tag{37}$$

Finally, using the fact that d ln $\psi = d\psi/\psi$, the full equation becomes a gradient,

$$\nabla\left\{\frac{\Phi}{m} - 2\mathscr{D}\nabla\left(\frac{i\,\partial\psi/\partial t + \mathscr{D}\Delta\psi}{\psi}\right)\right\} = 0. \tag{38}$$

This equation can now be easily integrated, to finally obtain a generalized Schrödinger-like equation (Nottale, 1993)

$$\mathscr{D}^2\Delta\psi + i\mathscr{D}\frac{\partial}{\partial t}\psi - \frac{\Phi}{2m}\psi = 0, \tag{39}$$

up to an arbitrary phase factor which may be set to zero by a suitable choice of the ψ phase.

Therefore the Schrödinger equation is the new form taken by the energy equation in the non-differentiable context. The standard Schrödinger equation of microphysics corresponds to the particular case $\mathscr{D} = \hbar/2m$, but the important point here is that all the physical–mathematical structure of the description is preserved for any constant value of the parameter $\mathscr{D}$, which therefore does not need to depend on the universal Planck constant $\hbar$. It is therefore possible for some particular systems to be described by such a Schrödinger-type equation, in terms of a parameter $\mathscr{D}$ which would be characteristic of this system (e.g., as a self-organization constant).

Arrived at that point, several steps have been already made toward the final identification of the function ψ with a wave function. Indeed, it is complex, solution of a Schrödinger equation, so that its linearity is also ensured (namely, if ψ_1 and ψ_2 are solutions, $a_1\psi_1 + a_2\psi_2$ is also a solution). We shall now complete the proof by showing that it fundamentally describes a wave, defined by a generalized Einstein–deBroglie wavelength, see below (Nottale, 1993, 2008; Célérier and Nottale, 2004), and that it is ultimately a wave of probability, since Born's postulate, according to which the probability of presence is given by the square of its modulus, can be derived from first principles in the scale relativity framework (Célérier and Nottale, 2004; Nottale, 2008).

2.3.4. *Fundamental wavelengths*

Let us first briefly recall how quantum mechanics is, from its origin, fundamentally a wave mechanics. Historically, this led to a very profound unification of matter and radiation by Einstein and de Broglie. In 1905, while the main view about light was that it was an electromagnetic wave, Einstein introduced, after Planck's work, the concept of a quantum of light which later became known as the photon. He related the

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frequency v of the wave to the energy E of the photon by the relation $E = hv$. This means that its wavelength is related to its momentum as $\lambda = h/p$, where h is Planck's constant (often replaced by $\hbar = h/2\pi$: h corresponds to a full rotation of the phase by an angle of 2π radiants, and therefore $\hbar$ to a rotation of 1 radiant). Later, in 1923, de Broglie made the same suggestion for matter, which was considered since the ancient Greeks as being made of "atoms" (today's particles). He proposed that matter also had a wave nature, the wavelength of this wave being linked to the classical momentum by the relation $\lambda_{\mathrm{deB}} = h/p$, and its period to energy as $\tau_{\mathrm{deB}} = h/E$.

In rest frame, the energy being $E = mc^2$, the de Broglie length becomes the so-called Compton length $\lambda_c = \hbar/mc$ and $\tau_c = \hbar/mc^2$. Such a length plays a very important role in quantum mechanics: ultimately, it yields a new definition of the mass of a particle, since it can be reduced to the mass up to fundamental constants ($\hbar$ and c).

The Einstein–de Broglie proposal is the origin of the wave mechanics that became quantum mechanics. However, up to now, it has not been understood from more fundamental first principles, since it is part of the fundamental axioms of quantum mechanics. As we have seen above, the scale relativity theory brings a new light on this question. Indeed, the construction of the function ψ and the demonstration of the Schrödinger equation of which it is a solution comes together with the derivation of the relation $S_0 = 2m\mathcal{D}$.

As we shall now see, this relation is nothing but a generalized Compton relation that moreover gives a geometric meaning to the Compton and de Broglie wavelengths. Indeed, in the case of standard quantum mechanics, S_0 is nothing but the universal action constant $\hbar$, while $\mathcal{D}$ defines the amplitude of the fractal fluctuations. Therefore the relation $\hbar = 2m\mathcal{D} = m\langle \mathrm{d}\xi^2\rangle/\mathrm{d}t$ provides a geometric interpretation of the Planck constant, which becomes a property of the fractal space.

Now the parameter $\mathcal{D}$ can also be expressed in terms of a length scale, $\lambda = 2\mathcal{D}/c$. Therefore, the relation $S_0 = 2m\mathcal{D}$ becomes a relation between the mass and a geometric quantity, which writes

$$\lambda_c = \frac{\hbar}{mc}. \tag{40}$$

We recognize here the definition of the Compton length, from which the de Broglie length can now be easily recovered. Indeed, the fractal fluctuation is a differential element of order $\frac{1}{2}$, i.e., it reads $\langle \mathrm{d}\xi^2\rangle = \hbar\,\mathrm{d}t/m$ in function of $\mathrm{d}t$, and then $\langle \mathrm{d}\xi^2\rangle = \lambda_x\,\mathrm{d}x$ in function of the space differential element, in which a length scale λ_x must be, once again, introduced for dimensional reasons. This implies $\lambda_x = \hbar/mv$, which is the non-relativistic expression for the de Broglie length. Then the full elementary displacements on the fractal space read $\mathrm{d}X = \mathrm{d}x + \sqrt{\lambda_x\,\mathrm{d}x}$ (and similar relations for the other coordinates), which may be written as

$$\mathrm{d}X = \mathrm{d}x\left[1 + \left(\frac{\lambda_x}{\mathrm{d}x}\right)^{1/2}\right] = \mathrm{d}x\left(1 + \frac{\lambda_x}{\langle \mathrm{d}\xi^2\rangle^{1/2}}\right). \tag{41}$$

One recognizes here the typical expression of a fractal behaviour including a transition to effective scale independence toward the large scales studied in the first companion paper. Therefore the de Broglie length, under its standard quantum form $\hbar/mv$, and under its generalized form $2\mathcal{D}/v$, can be readily identified with such a fractal to non-fractal transition.

Therefore a geometric meaning has now been given to these fundamental lengths of quantum mechanics, and, by extension, to the mass itself (since, modulo the fundamental constants $\hbar$ and c, the Compton length is the inverse of the inertial mass). We note that these length scales are to be understood as geometric structures of scale space (more generally space–time), not of standard space.

The last step for a complete identification of a quantum-type tool (in a generalized way which no longer relies only on the universal constant $\hbar$) amounts to recover the fundamental meaning of the wave function as a wave of probability (this is Born's postulate). This will be made by obtaining a third representation of the motion equation under a hydrodynamics-like form.

2.3.5. Fluid mechanics form of the motion equations

We have given above two representations of the fundamental equations of dynamics in a fractal and locally irreversible context. The first representation is the equation of geodesics that is written in terms of the complex velocity field, $\mathcal{V} = V - \mathrm{i}U$. The second representation is the Schrödinger equation, whose solution is a wave function ψ. Both representations are related by the transformation

$$\mathcal{V} = -2\mathrm{i}\mathcal{D}\nabla \ln \psi. \tag{42}$$

Let us decompose the wave function in terms of a modulus $A = \sqrt{P}$ and of a phase θ, namely, we write the wave function under the form $\psi = \sqrt{P} \times \mathrm{e}^{\mathrm{i}\theta}$. We shall now build a mixed representation, in terms of the real part of the complex velocity field, V, and of the square of the modulus of the wave function, P. This representation has the advantage to re-connect the description to the initial scale-relativistic view of motion as following a fluid of fractal geodesics. We shall indeed obtain, as we shall now see, a fluid mechanics-type description of the classical part of the velocity field V, but with an added quantum potential which profoundly changes the meaning and behaviour of this description.

By separating the real and imaginary parts of the Schrödinger equation and by making the change of variables from ψ to (P, V), we obtain, respectively, a generalized Euler–Newton-like equation and a continuity-like equation (Nottale et al., 2000a) (see the proof in Appendix B)

$$\left(\frac{\partial}{\partial t} + V \cdot \nabla\right) V = -\nabla\left(\Phi - 2\mathcal{D}^2 \frac{\Delta\sqrt{P}}{\sqrt{P}}\right), \tag{43}$$

$$\frac{\partial P}{\partial t} + \mathrm{div}(PV) = 0. \tag{44}$$

This system of equations is equivalent to the classical system of equations of fluid mechanics (with no pressure and no vorticity), except for the appearance of an extra potential energy term Q that writes

$$Q = -2\mathcal{D}^2 \frac{\Delta\sqrt{P}}{\sqrt{P}}. \tag{45}$$

The existence of this potential energy is, in the scale relativity approach, a very manifestation of the geometry of space, namely, of its non-differentiability and fractality (Nottale, 1998a, 2005; Nottale et al., 2000a) in similarity with Newton's potential being a manifestation of curvature in Einstein's general relativity framework. It is a generalization of the quantum potential (Madelung, 1927; Bohm, 1952) of standard quantum mechanics. However, its nature was misunderstood in this framework, since the variables V and P were constructed from the wave function, which is set as one of the axiom of quantum mechanics, as the Schrödinger equation itself. On the contrary, in the scale relativity theory, we know from the very beginning of the construction that V represents the velocity field of the fractal geodesics, and the Schrödinger equation is derived from the very equation of these geodesics.

Born's postulate, according to which the square of the modulus of the wave function $P = |\psi|^2$ gives the probability of presence of the particle, can now be inferred from the scale relativity construction.

Indeed, we have identified the wave-particle with the various geometric properties of a subset of the fractal geodesics of a non-differentiable space–time. In such an interpretation, a measurement (and more generally any knowledge about the system) amounts to a selection of the subsample of the geodesics family in which are kept only the geodesics having the geometric properties corresponding to the measurement result. Therefore, just after the measurement, the system is in the state given by the measurement result, in accordance with von Neumann's postulate of quantum mechanics. (It is remarkable that the *selection* aspect of this new interpretation of the measurement in quantum mechanics is close to similar essential concepts in life sciences, as e.g., in species evolution.)

As a consequence, the probability for the particle to be found at a given position must be proportional to the density of the fluid of geodesics. We already know its velocity field, which is expected to be given by $V(x(t), t)$, identified, at the classical limit, with a classical velocity field. The density ϱ of the geodesics has not yet been introduced at this level of the construction (contrarily to most stochastic approaches where it is introduced from the very beginning and is used to define averages). In order to calculate it, we remark that it is expected to be a solution of a fluid-like Euler and continuity system of equations, namely,

$$\left(\frac{\partial}{\partial t}+V\cdot\nabla\right)V=-\nabla(\Phi+Q), \tag{46}$$

$$\frac{\partial\rho}{\partial t}+\mathrm{div}(\varrho V)=0, \tag{47}$$

where Φ describes an external scalar potential possibly acting on the fluid, and Q is the potential that is expected to appear as a manifestation of the fractal geometry of space (in analogy with the appearance of the Newtonian potential as a manifestation of the curved geometry in general relativity). This is a system of four equations (since Eq. (46) is vectorial and is therefore made of three equations) for four unknowns, (ϱ, V_x, V_y, V_z), that would be therefore completely determined by such a system.

Now these equations are exactly the same as Eqs. (43), (44), except for the replacement of the square of the modulus of the wave function P by the fluid density ϱ. Therefore this result allows one to univoquely identify $P=|\psi|^2$ with the probability density of the geodesics, i.e., with the probability of presence of the particle (Nottale, 2008). Moreover, one identifies the non-classical term Q with the new potential which is expected to emerge from the fractal geometry (Nottale, 2005). Numerical simulations in which the expected probability density can be obtained directly from the distribution of geodesics without writing the Schrödinger equation, have confirmed this result (Hermann, 1997).

We have recently proposed to obtain a new kind of quantum-like macroscopic superfluid in a laboratory experiment by applying such a generalized quantum potential to a classical fluid through a retro-active loop involving real time measurements of its density (Nottale, 2008; Nottale and Lehner, 2006). Numerical simulations of such an experiment have given encouraging results (Nottale and Lehner, 2006).

2.3.6. Relation to diffusion processes

2.3.6.1. Motivation. By writing the equations of motion under the form of a macroscopic Schrödinger equation, we have actually obtained a theory of self-organization.

Indeed, as we shall show in more detail in Section 2.4 by giving some explicit examples, a Schrödinger-type equation is characterized by the existence of stationary solutions yielding well-defined peaks of probability linked to quantization laws, themselves a consequence of the limiting (or environmental) conditions, of the forces applied and of the symmetries of the system. It has therefore been suggested (Nottale, 1997) that these peaks of probability density can be interpreted as a tendency for the system to make structures. In other words, the theory is able to predict, not a fully deterministic organization, but rather the most probable structures among the infinity of other close possibilities. Such a feature of this approach is compatible with the large variability that characterizes living systems. Moreover, it is a genuine theory of self-organization, since the structuring comes from the interior instead of being a consequence of an external force (but the particular organization obtained in a given situation is related to the environmental conditions). Namely, the natural tendency for the system to self-organize comes from the quantum nature of the laws, which is itself, in the scale relativity framework, a manifestation of relativity and covariance (i.e., of the non-absolute existence of the various properties defining a physical or biological system).

Now an important question concerning this formalism, given its widespread use in modelling of chemical reactions and biological systems, is its relation to diffusion processes. It could seem paradoxical that, starting from a description of infinitesimal displacements made in terms of stochastic processes, one obtains in the end a theory of self-organization.

We shall in this section solve this paradox by showing that the principle of relativity (under its form of geodesic principle) has actually recombined the initial twin velocity process in terms of a genuine anti-diffusion. This means that the self-organization properties of the scale relativity theory lead to an entropy decrease, i.e., to a neguentropy, the source of which has been unsuccessfully sought for a long time as the foundation of life sciences.

2.3.6.2. Diffusion approach to quantum laws. The question of the relation between quantum laws and diffusion processes has been asked for a long time. Indeed, there is a formal analogy between the classical diffusion equation

$$\mathscr{D}\Delta P - \frac{\partial P}{\partial t} = 0 \tag{48}$$

and the quantum, free Schrödinger-type equation,

$$\mathscr{D}\Delta\psi + \mathrm{i}\frac{\partial \psi}{\partial t} = 0 \tag{49}$$

(where $\mathscr{D} = \hbar/2m$ in standard quantum mechanics). Moreover, the parallelism between the undeterminisms of the quantum mechanical behaviour and of stochastic processes has led many authors to attempt to understand the quantum behaviour in terms of real particle trajectories and classical diffusion processes (de Broglie, 1926; Bohm, 1952; Fenyes, 1952; Weizel, 1953; Bohm and Vigier, 1954; Nelson, 1966).

Actually, it has been shown that the various attempts to recover the quantum behaviour in terms of classical diffusion processes have a fundamental flaw (Grabert et al., 1979; Wang and Liang, 1993; Nottale, 1997; Nottale et al., 2000a). Namely, it is necessary to introduce two (forward and backward) diffusion processes (because of the complex numbers nature of quantum mechanics), and one of them can be shown to actually have no physical meaning. (Note that, in the scale relativity approach, the elementary description is made in terms of a twin stochastic process, but it is not associated with a diffusion interpretation, since it is understood as a direct manifestation of the non-differentiable geometry).

But the differences in the behaviours of their solutions are by far larger than their similarity: they can even be considered as exactly opposite. Indeed, while diffusion processes in open space are the archetype for dissipative, non-stationary, non-isoentropic systems characterized by disorganization (typically, entropy increase proportional to time t and spreading proportional to $\sqrt{t}$), the Schrödinger equation exhibits, for given conditions of limits, field and symmetry, stationary and quantized solutions which can be viewed as an archetype for self-organization (e.g., atoms). A typical example of such opposite behaviours is the diffusion in a box, which leads to an unstructured final state of constant density, while the corresponding Schrödinger process yields well-defined structures given by peaks of probability density.

Let us analyse in more detail the reason for this opposite behaviour and its consequences for applications to living systems.

2.3.6.3. Diffusion potential. As shown in Appendix C, by applying to a diffusion process (cf Eq. (48))

$$\frac{\partial P}{\partial t} = \mathscr{D}\Delta P, \tag{50}$$

the change of variable

$$V = v - \mathscr{D}\nabla \ln P, \tag{51}$$

one finds that the new velocity field thus constructed is a solution of the standard continuity equation (cf. Eq. (44)),

$$\frac{\partial P}{\partial t} + \mathrm{div}(PV) = 0, \tag{52}$$

and of a Euler equation that reads (see Appendix B)

$$\left(\frac{\partial}{\partial t} + V.\nabla\right)V = -2\mathscr{D}^2\nabla\left(\frac{\Delta\sqrt{P}}{\sqrt{P}}\right). \tag{53}$$

We have therefore obtained exactly the fluid mechanics form of the generalized Schrödinger equation, except for the fundamental difference that the quantum potential $Q = -2\mathscr{D}^2\Delta\sqrt{P}/\sqrt{P}$ has been replaced by its opposite, $Q_{\mathrm{diff}} = +2\mathscr{D}^2\Delta\sqrt{P}/\sqrt{P}$, where $\mathscr{D}$ is now the diffusion coefficient.

This result is remarkable for several reasons:

(i) it gives an equivalence between a standard fluid subjected to a force field and a diffusion process. However, this force is very particular, since it is expressed in terms of the probability density at each point and instant;

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(ii) the above "diffusion force" $F_{\text{diff}} = -2\mathscr{D}^2\nabla(\Delta\sqrt{P}/\sqrt{P})$ derives from a potential Q_{diff}. This expression introduces, in a striking way, a square root of probability in the description of what remains a totally classical diffusion process (while usually one encounters probabilities P in classical mechanics and probability amplitudes $\sqrt{P} \times \mathrm{e}^{\mathrm{i}\theta}$ in quantum mechanics);
(iii) but there is more: not only is it expressed in terms of the square root of probability, but this "diffusion potential" is exactly the opposite of the generalized quantum potential $Q = -2\mathscr{D}^2\Delta\sqrt{P}/\sqrt{P}$.

The relation between quantum mechanics and diffusion processes is now enlightened in a new way: they appear as exactly opposite. We conclude that it is therefore impossible to obtain quantum mechanics from a standard classical diffusion process, since it is now clear that quantum mechanics corresponds to an "anti-diffusion" rather than a diffusion. It also illuminates in a new way the relation between diffusion laws, which are an archetype for laws of disorganization and entropy increase following the second principle of thermodynamics (it increases directly proportionally to time), and quantum-type laws, which can therefore be seen as an archetype for laws of self-organization and local entropy decrease (neguentropy).

This fundamental result can be illustrated by performing numerical simulations of the Euler and continuity system of fluid mechanics equations in three cases (Nottale and Lehner, 2006):

(i) applying the additional diffusion potential $Q_{\text{diff}} = +2\mathscr{D}^2\Delta\sqrt{P}/\sqrt{P}$;
(ii) applying no potential in addition to the externally applied ones;
(iii) applying the additional quantum potential $Q = -2\mathscr{D}^2\Delta\sqrt{P}/\sqrt{P}$.

In such numerical simulations, one can continuously change the value of the potential amplitude from $-2\mathscr{D}^2$ to $2\mathscr{D}^2$ (contrarily to the case of standard quantum mechanics where it is fixed to the value $\hbar^2/2m$). This allows one to have a continuous transition from a diffusive, disorganized, entropy increasing system, to a classical system (which can organize itself or disorganize itself depending on the conditions in a causal way), then to a quantum-type superfluid-like system characterized by a spontaneous self-organization.

2.3.7. Generalization of quantum laws

The generalized Schrödinger form of the motion equation

$$\mathscr{D}^2\Delta\psi + \mathrm{i}\mathscr{D}\frac{\partial}{\partial t}\psi - \frac{\Phi}{2m}\psi = 0 \tag{54}$$

has been obtained in the most simple case of an underlying scale law given by a constant fractal dimension $D_{\mathrm{F}} = 2$. It is equivalent to the standard Schrödinger equation, except for the replacement of $\hbar$ by $2m\mathscr{D}$, which does not changes the mathematical structure of the equation and of its solutions.

However, we have seen in the companion paper that this scale law is the equivalent for scale of what inertia is for motion, so that it can be generalized to new laws of scale dynamics (including, e.g., log-periodic fluctuations, non-linearity, etc.). These generalized scale laws (including constant fractal dimensions different from 2), that describe the inner fractal structures of "points", are expected to yield a new generalization of the form of the equations of motion of such structured points.

A simple way to obtain this newly generalized Schrödinger-type equation (Nottale, 1995, 1996a) amounts to use the $D_{\mathrm{F}} = 2$ scale-inertial case as a reference case. All the new scale laws can be characterized by a scale-dependent fractal dimension $D_{\mathrm{F}} = D_{\mathrm{F}}(\varepsilon)$, or equivalently by a variable djinn $\tau(\varepsilon) = D_{\mathrm{F}}(\varepsilon) - D_{\mathrm{T}}$. In the case of the differential calculus in a fractal space, we identify the scale variable ε with the differential $\mathrm{d}t$, and we write the average fractal fluctuation under the form, which generalizes Eq. (5) (with a different definition of the parameter $\mathscr{D}$)

$$\langle \mathrm{d}\xi^2 \rangle = 2\mathscr{D}\,\mathrm{d}t^{2/D_{\mathrm{F}}(\mathrm{d}t)} = \{2\mathscr{D}\,\mathrm{d}t^{(2/D_{\mathrm{F}}(\mathrm{d}t))-1}\}\,\mathrm{d}t = 2\tilde{\mathscr{D}}(\mathrm{d}t)\,\mathrm{d}t. \tag{55}$$

It therefore keeps the previous $D_{\mathrm{F}} = 2$ form, but now in terms of a scale-dependent coefficient $\tilde{\mathscr{D}}(\mathrm{d}t) = \mathscr{D}\,\mathrm{d}t^{[2/D_{\mathrm{F}}(\mathrm{d}t)]-1}$. The whole derivation of the Schrödinger equation is preserved, so that one obtains

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the new generalization (Nottale, 1995, 1996a)

$$\tilde{\mathscr{D}}^2(\mathrm{d}t)\,\Delta\psi + \mathrm{i}\tilde{\mathscr{D}}(\mathrm{d}t)\frac{\partial}{\partial t}\psi - \frac{\Phi}{2m}\psi = 0. \tag{56}$$

This result demonstrates the critical nature of $D_{\mathrm{F}} = 2$, since it is only for this value that the Schrödinger-type equation of motion takes its standard form in which there is no explicit scale dependence. From the viewpoint of this more general equation, one discovers that it is generally scale-dependent (i.e., the differential element $\mathrm{d}t$ does not disappear from the equation), except for the particular case $D_{\mathrm{F}} = 2$. Now, even in this case, though the scale dependence is not explicit in the equation, it is, however, a hidden property since it reappears in the solutions under the form of the Heisenberg relations, which relate the space–time scales to the momentum-energy scales, namely, $\delta p \times \delta x \approx \hbar$ and $\delta E \times \delta t \approx \hbar$.

According to a recent result, the previous derivation of the Schrödinger equation remains effective when working in terms of the full velocity field, i.e., keeping not only the classical (differentiable) part of the velocity field, but also the fractal (non-differentiable) part (Nottale, 2008). In this general case, the wave function is still related to the full complex velocity field by the relation $\tilde{\mathscr{V}} = -2\mathrm{i}\mathscr{D}\nabla\ln\psi$. Now, as we have stressed at the beginning of this paper, the main scale relativity tool consists of describing non-differentiable functions as fractal functions, i.e., explicitly resolution-dependent functions. Therefore the full velocity field is in this case an explicit function of the time-differential $\mathrm{d}t$, i.e., $\tilde{\mathscr{V}} = \tilde{\mathscr{V}}(x, t, \mathrm{d}t)$. As a consequence, the same is true of the wave function. In other words, this means that we generally expect, from the scale relativity theory, the general solutions of the Schrödinger equation to be non-differentiable (Nottale, 2008), i.e., to be explicitly scale-dependent and divergent fractal wave functions, $\psi = \psi(x, t, \mathrm{d}t)$. This result had been anticipated in the framework of standard quantum mechanics by the discovery of non-differentiable solutions to quantum mechanical equations by Berry (1996) and Hall (2004).

The above newly generalized Schrödinger equation can now take fully its sense: the explicit dependence of the coefficient $\tilde{\mathscr{D}}$ on the scale $\mathrm{d}t$ is no longer problematic, since the solution ψ is expected to be itself an explicit function of $\mathrm{d}t$.

2.4. *Application to biological processes*

2.4.1. *Conditions of application*

Scale relativity extends the potential domain of application of Schrödinger-like equations to Newtonian dynamical systems in which the three conditions that lead to its derivation, namely, (i) infinite or very large number of trajectories, (ii) fractal dimension 2 of individual trajectories, (iii) local irreversibility, are fulfilled. These conditions apply in an exact way in the microphysical case leading to the standard quantum mechanics based on the microscopic universal Planck constant $\hbar$, but they may also apply in macroscopic systems where they are fulfilled in an approximate way in terms of a macroscopic constant (Nottale, 1993, Chapter 7.2). Such systems are typically those where the information on the angles, on the position and on time are lost, which includes chaotic systems on long time-scales, beyond their horizon of predictability (typically 10–20 times the inverse of the Lyapunov exponent).

Let us consider indeed a strongly chaotic system, such that the gap between any couple of trajectories diverges exponentially with time. In the reference frame of one trajectory, which we describe as uniform motion on the z-axis, namely, $x = 0$, $y = 0$, $z = at$, a second trajectory is then described by the equations

$$x = \delta x_0(1 + \mathrm{e}^{t/\tau}), \quad y = \delta y_0(1 + \mathrm{e}^{t/\tau}), \quad z = at + \delta z_0(1 + \mathrm{e}^{t/\tau}), \tag{57}$$

where we have assumed a single Lyapunov exponent $1/\tau$ for simplicity of the argument. By eliminating the time between these equations, they become (Nottale, 1993, 1996a)

$$y = \frac{\delta y_0}{\delta x_0}x, \quad z = \frac{\delta z_0}{\delta x_0}x + a\tau\ln\left(\frac{x}{\delta x_0} - 1\right). \tag{58}$$

Such an $(x + \ln x)$ type behaviour describes a sudden change of direction (see Nottale, 1993, Fig. 7.3) which looks quite like a scattering effect, and even like a collision when it is seen with a time resolutions $\Delta t \gg \tau$. At long time-scales it therefore becomes apparently non-differentiable, with two different slopes. Moreover, the final

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direction of the trajectory in space is given by the initial "uncertainty vector" $\varepsilon = (\delta x_0, \delta y_0, \delta z_0)$ which is uncontrollable (it may take its origin at the quantum scales themselves), so that the second trajectory can emerge with any orientation with respect to the first. Since such breaking points in the slopes occur everywhere, one finally obtains a Markov–Wiener process of the Brownian motion kind which is of fractal dimension $D_F = 2$.

This does not mean that the real Brownian motion of Brown and other particles could come under the scale relativity description, for two reasons. The first is that the dynamics is in this case a Langevin dynamics where a force yields a velocity, instead of the acceleration of Newtonian dynamics. The second is that, as can be seen in Section 3.2.5 and Fig.5 of the first of these companion papers, the fractal dimension 2 fluctuation law $\langle d\xi^2 \rangle = 2\mathscr{D}\, dt$, which underlies the derivation of the Schrödinger equation, must be valid on a large enough range of scales, of at least 10^4–10^5.

One may nevertheless consider that the three above general conditions, plus the two additional ones (Newtonian dynamics and large enough range of scales with fractal geometry) be well adapted to the description of many aspects (although clearly not all aspects) of living systems.

Macroscopic Schrödinger equations can therefore be constructed, which are not based on Planck's constant $\hbar$, but on constants that are specific of each system and may emerge from their self-organization.

2.4.2. Morphogenesis

A generalized Schrödinger equation can be viewed as a fundamental equation of morphogenesis. Let us indeed give a simple example of such an application to the emergence of flower-like shapes as a manifestation of its solutions (Nottale, 2001, 2004).

In living systems, morphologies are acquired through growth processes. One can attempt to describe such a growth in terms of an infinite family of virtual, fractal and locally irreversible trajectories. Their equation can therefore be written under the form of a geodesic, free motion-like equation, then it can be integrated in terms of a Schrödinger equation (39).

We now look for solutions describing a growth from a centre. This problem is formally identical to that of planetary nebulae (da Rocha and Nottale 2003a, b), which are stars that eject their outer shells, and, from the quantum point of view, to the problem of particle scattering. The solutions looked for correspond to the case of the outgoing spherical probability wave.

Depending on the potential, on the boundary and on the symmetry conditions, a large family of solutions can be obtained. Let us consider here only the simplest ones, i.e., central potential and spherical coordinates. The probability density distribution of the various possible values of the angles is given in this case by the spherical harmonics:

$$P(\theta, \varphi) = |Y_{lm}(\theta, \varphi)|^2. \tag{59}$$

These functions show peaks of probability for some angles, depending on the quantized values of the square of angular momentum L^2, characterized by the quantum number l and of its projection L_z on axis z, characterized by the quantum number m. In other words, L^2 and L_z can take only some particular values proportional to these quantum numbers, which are integers, and the corresponding morphologies are themselves quantized. Namely, instead of having a continuum of possible morphologies, only some peculiar morphologies are possible (see da Rocha and Nottale, 2003a, b).

Finally, a more probable morphology is obtained by making the structure grow along angles of maximal probability. The solutions obtained in this way, show floral "tulip"-like shape (da Rocha and Nottale, 2003a, b). Now the spherical symmetry is broken in the case of living systems. One jumps to discrete cylindrical symmetry: this leads in the simplest case to introduce a periodic quantization of angle θ (measured by an additional quantum number k), which gives rise to a separation of discretized "petals". Moreover there is a discrete symmetry breaking along axis z linked to orientation (separation of "up" and "down" due to gravity, growth from a stem). This results in floral-like shapes such as those in Figs. 2 and 3 (which gives the successive images of an animation modelling the opening of such a shape).

2.4.3. Formation, duplication and bifurcation

A fundamentally new feature of the scale relativity approach with regard to problems of formation is that the Schrödinger form taken by the geodesic equation can be interpreted as a general tendency to make

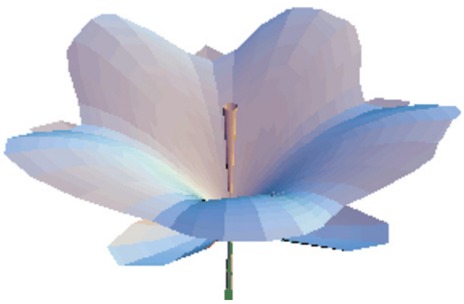

Fig. 2. This is not a flower, but a representation of the maximum probability density $P = |\psi|^2$, where ψ is solution of a Schrödinger equation that describes a growth process from a centre ($l = 5, m = 0$). The "petals", "sepals" and "stamen" are traced along angles of maximal probability density and are all given by the same wave function. A quantization of the angle θ that gives an integer number of "petals" (here, $k = 5$) has been added, and a constant gravity-like force involving an additional curvature of "petals".

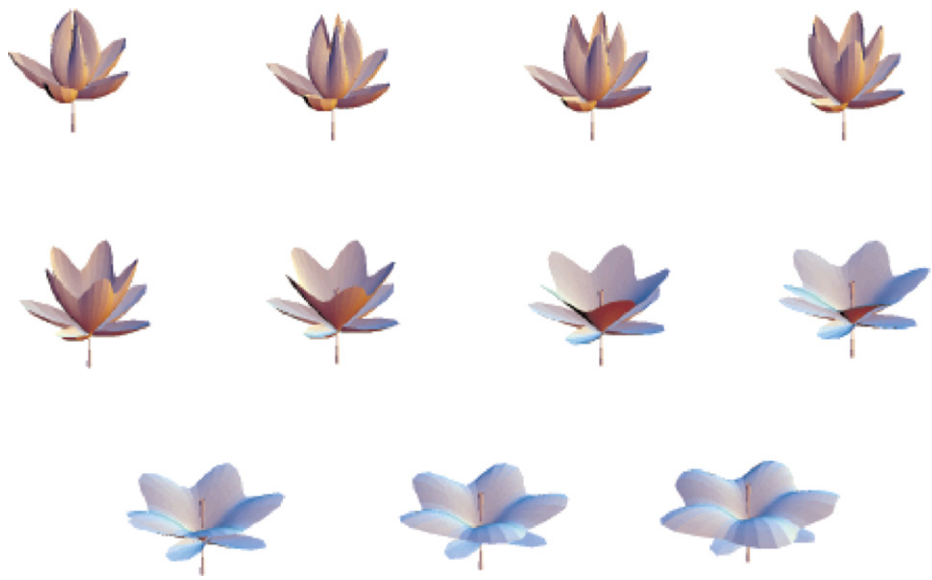

Fig. 3. Morphogenesis of a flower-like structure, solution of a time-dependent Schrödinger equation that describes a growth process from a centre ($l = 5, m = 0$). The "petals", "sepals" and "stamen" are traced along angles of maximal probability density. The force of "tension" is now variable, simulating the opening of the "flower". (Steps of an animation, from left to right and up to down.)

structures, i.e., to self-organization. In the framework of a classical deterministic approach, the question of the formation of a system is always posed in terms of initial conditions. In the new framework, structures are formed whatever the initial conditions, in correspondence with the field, the boundary conditions and the symmetries, and in function of the values of the various conservative quantities that characterize the system.

A typical example is given by the formation of gravitational structures from a background medium of strictly constant density. This problem has no classical solution: no structure can form and grow in the absence of large initial fluctuations. On the contrary, in the present quantum-like approach, the stationary Schrödinger equation for an harmonic oscillator potential (which is the form taken by the gravitational potential in this case) does have confined stationary solutions characterized by a quantized energy $E_n = (2n + 3)m\mathscr{D}\omega$, where n is an integer quantum number that can take values from 0 to infinity and where ω is the proper frequency of the oscillations. The fundamental level or vacuum solution (the vacuum is the state of minimal energy), defined by the quantum number $n = 0$, is made of one-body structure with Gaussian distribution, the second level ($n = 1$) is a two-body structure (see Fig. 4), then one obtains chains, trapezes, etc. for higher levels. It is remarkable that, whatever the scales in the large-scale Universe (stars, clusters of stars, galaxies, clusters of

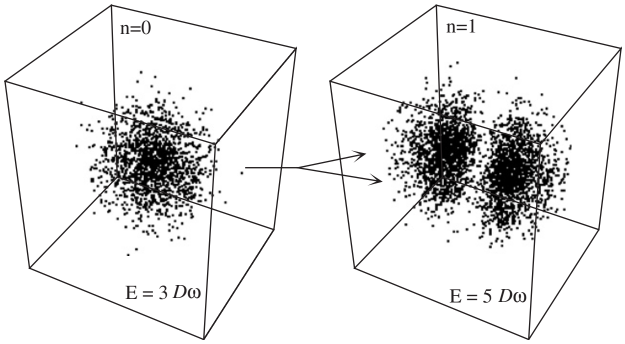

Fig. 4. Model of duplication. The stationary solutions of the Schrödinger equation in a 3D harmonic oscillator potential can take only discretized morphologies in correspondence with the quantized value of the energy. The energy of the fundamental level (left, one-body structure) is given by $E_0 = 3\mathcal{D}\omega$, where ω is the proper frequency and $\mathcal{D}$ the fundamental parameter of the theory defined by the amplitude of the fractal fluctuations. If one increases the energy from this value, no stable solution can exist before it reaches the second quantized level at $E_1 = 5\mathcal{D}\omega$ (right, two-body structures). The solutions of the time-dependent equation show that the system spontaneously jumps from the one-body structure to the two-body structure morphology, provided the jump in energy takes the quantized value $2\mathcal{D}\omega$.

galaxies) the zones of formation show in a systematic way this kind of double, aligned or trapeze-like structures (Nottale, 1996a, 2001; da Rocha and Nottale, 2003a).

These solutions may also be meaningful in other domains than gravitation, because the harmonic oscillator potential is encountered in a wide range of conditions. It is the general force that appears when a system is displaced from its equilibrium conditions, and, moreover, it describes an elementary clock. For these reasons, it is well adapted to an attempt to describe living systems, at first in a rough preliminary way.

Firstly, such an approach could allow one to ask the question of the origin of life in a renewed way. This problem is the analog of the vacuum (lowest energy) solutions, i.e., of the passage from a non-structured medium to the simplest, fundamental level structures. Provided the description of the prebiotic medium comes under the three basic conditions that lead to the emergence of a macroscopic quantum-type mechanics (namely, complete information loss on angles, position and time), we suggest that it could be subjected to a Schrödinger equation (with a coefficient $\mathcal{D}$ self-generated by the system itself). Such a possibility is supported by the formal structure of thermodynamics (Peterson, 1979), which can be formulated in terms of Noether's theorem, action principle and Euler–Lagrange equations in the same manner as mechanics. One can then consider a future application of all the scale-relativistic methods to thermodynamics, and therefore to chemistry, this possibly leading to a new form of macroscopic quantum-like description of these domains. Such a description, as we have shown, would have powerful properties of self-organization, morphogenesis and spontaneous structure formation, and could be applied to the basic mechanisms of prebiotic systems, such as polymer growth, etc. But clearly much cross-disciplinary work is needed in order to implement such a working program.

Secondly, the passage from the fundamental level to the first excited level provides us with a (rough) model of duplication (see Nottale, 2001, 2004, and Figs. 4, 5 and 6). Once again, the quantization implies that, in case of energy increase, the system will not increase its size, but will instead be led to jump from a single structure to a double structure, with no stable intermediate step between the two stationary solutions $n = 0$ and 1. Moreover, if one comes back to the level of description of individual trajectories, one finds that from each point of the initial one-body structure there exist trajectories that go to the two final structures. We expect, in this framework, that duplication needs a discretized and precisely fixed jump in energy.

Such a model can also be applied to the description of a branching process (Fig. 6), e.g., in the case of a tree growth when the previous structures remain and add themselves along a z-axis instead of disappearing as in cell duplication.

2.4.4. *Prediction of biological sizes*

One of the main interests of the new macroscopic quantum-type approach is its capacity to make predictions about the size of the structures which are formed from its self-organizing properties.

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In some cases, this prediction depends only on the boundary conditions, i.e., on the environment (in a biological context). Consider for example the free geodesics in a limited region of space. The classical fluid equation would yield a constant probability density (i.e., no structure), while the scale relativity description yields an equation similar to the Schrödinger equation for a particle in a box, which is solved in one dimension in terms of a probability density

$$P = |\psi|^2 = \frac{2}{a} \sin^2 \frac{\pi n}{a} x. \tag{60}$$

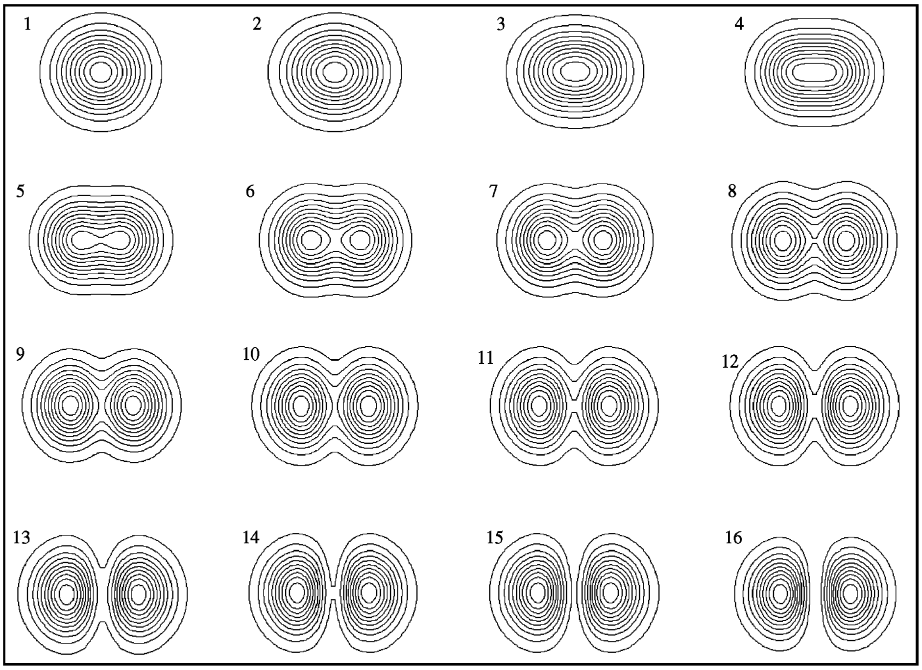

Fig. 5. Model of duplication. Successive solutions during time evolution of the time-dependent Schrödinger equation in a 3D harmonic oscillator potential with varying energy, showing how the system jumps from the one-body structure to the two-body structure morphology when the jump in energy takes the quantized value $2\mathcal{D}\omega$. The successive figures give the isovalues of the density of probability for 16 time steps.

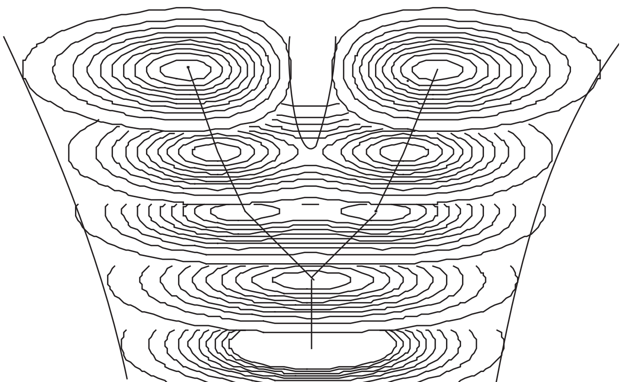

Fig. 6. Model of branching/bifurcation. Successive solutions of the time-dependent 2D Schrödinger equation in an harmonic oscillator potential are plotted as isodensities. The energy varies from the fundamental level ($n = 0$) to the first excited level ($n = 1$), and as a consequence the system jumps from a one-body structure to a two-body structure morphology. In this case of branching/tree-like growth, the preceding structures remain (i.e., the time and z-axis variables are similar, contrarily to the cell division-like case).

The multidimensional case is a product of similar expressions for the other coordinates. One therefore obtains, in the fundamental level ($n = 1$), a peaked structure whose typical size is given, e.g., by its dispersion

$$\sigma_x = \frac{a}{2\pi}\sqrt{\frac{\pi^2}{3} - 2} \approx 0.1807a, \tag{61}$$

and therefore depends only on the size of the box (see Fig. 7).

In other cases, the size depends on the fluctuation parameter $\mathcal{D}$, which generalizes the ratio $\hbar/2m$ of standard quantum mechanics. For example, in the harmonic oscillator solutions considered in Figs. 4–6, the dispersion of the Maxwellian probability density distribution of the fundamental level is given by $\sigma^2 = \hbar/m\omega$, where ω is the proper frequency of oscillations, i.e., in the generalized case,

$$\sigma = \sqrt{\frac{\mathcal{D}}{\omega}}. \tag{62}$$

One could think, in these conditions, that a genuine theoretical prediction of the typical sizes of biological systems could not be done in the absence of a theoretical prediction of the parameter $\mathcal{D}$ itself. However, this argument would also apply to atomic and molecular physics, in which the Planck constant $\hbar$ is not theoretically predicted either. This problem is solved in standard quantum physics by measuring it through many different experiments, then using the resulting value in new experiments, thus allowing a predictive power.

We contemplate the possibility to use the same method in biology, even though the value of $\mathcal{D}$ is no longer universal. Namely, for a given system, one expects the appearance of many different effects from such a macroscopic quantum-like theory, among other, interferences, quantization of energy, momentum, angular momentum, shapes, sizes, angles, etc., so that the constant $\mathcal{D}$ can be measured from any of these effects (e.g., the energy of the linear oscillator is $E_n/m = (2n + 1)\mathcal{D}\omega$) and then taken back to predict the size ($\sqrt{\mathcal{D}/\omega}$ for the linear oscillator) and other properties of the system under consideration. More generally, the typical sizes are expected to be of the order of the de Broglie length, given in the new case by $\lambda_{\mathrm{dB}} = 2\mathcal{D}/v$ for a linear motion of mean velocity v, and by the thermal de Broglie length $\lambda_{\mathrm{th}} = 2\mathcal{D}/\langle v^2\rangle^{1/2}$ for a medium or an ensemble of particles.

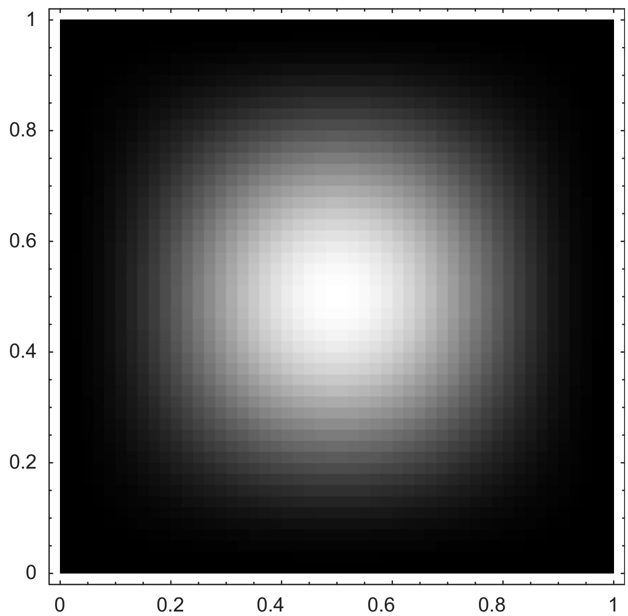

Fig. 7. Probability density for a free quantum particle in a box, in the fundamental level (stationary solution). A structure is formed, the size of which depends only on the size of the box (see text), while the classical distribution would yield a constant probability density, i.e., no structure.

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But we can do better, since, contrary to standard quantum mechanics in which $\hbar$ is a foundation constant, we have at our disposal a very definition of $\mathscr{D}$ from the underlying scale relativity theory. Namely, it is defined as the amplitude of mean fractal fluctuations,

$$\mathscr{D} = \frac{1}{2}\frac{\langle \mathrm{d}\xi^2 \rangle}{\mathrm{d}t}, \tag{63}$$

which means that it is defined exactly as a diffusion coefficient. This is also apparent in the equation for the v_+ velocity field (introduced in Section 2.2.3) which is nothing but a standard Fokker–Planck-like equation (for a probability density instead of a matter density), namely,

$$\frac{\partial P}{\partial t} + \mathrm{div}(P v_+) = \mathscr{D}\Delta P. \tag{64}$$

In other words, this means that what would have been classically described as a diffusion process in the absence of velocity doubling and explicit scale dependence becomes in the scale relativity theory a quantum-like process. We therefore suggest that the value of $\mathscr{D}$ is already known and measured as an apparent diffusion coefficient in many biological and biochemical systems, in particular in those that show definite self-organization despite this diffusion interpretation.

A last method of predicting typical biological scales will be given in Section 3, in terms of probability peaks for their ratio to relevant scales such as the atomic Bohr scale, or the maximum size of living systems implied by physical constraints on Earth.

2.4.5. Prospects: other quantum-type properties

Let us conclude this part by remarking that we have only given here a hint of the wide domain of possible applications of such a biological macroscopic quantum-type theory. As already remarked, not all properties of the microscopic standard quantum mechanics are expected to be recovered in the macroscopic theory. For example, the account of the EPR paradox in scale relativity (Célérier and Nottale, 2004, 2006) is based on the description of the space–time of microphysics as being fully non-differentiable (i.e., scale divergent), without any lower scale limit. In biology, the non-differentiability is just an approximation valid on a given large enough range of scales, but not at all scales, so that no EPR paradox is expected in this case. Another example is the indistinguishability of identical particles. In the scale relativity description of elementary particles, it is understood as a consequence of the purely geometric description of particles as being nothing else but the geodesics themselves of the fractal space–time. There is no longer any "particle" that would follow a "trajectory", but only an infinite bundle of purely geometric paths. The various properties (conservative quantities) of the "particles" are derived as internal geometric structures of the geodesics, so that there is no possibility to distinguish between subsamples of this bundle when the structures are the same. This is not the case in the application to biology, where one considers again "particles" that follow trajectories.

But many other features of the standard quantum theory are recovered in its macroscopic version, which may be relevant to the description of biological systems. Some of them have been briefly considered hereabove (quantization, non-dissipation, self-organization, confinement, structuration conditioned by the environment, etc.), but the list is long, so we intend to dedicate forthcoming papers to these questions.

Among them is the question of their evolutionary time. To deal with this problem, recall that we have derived not only the time-independent Schrödinger equation, which applies to stationary solutions, but also the time-dependent one, which describes the time evolution in this framework (see Fig. 5 for an example of solution of this time-dependent equation).

The influence of neighbouring systems is also a fundamental question for biology. As a first step, it can be included through the various symmetry and boundary conditions and through the intervention of an external mean field which would summarize their effect. But, as a second step, one may also, in a more complete way, write a complete system of equations including the effects of fractality and of non-differentiability, and describing the system, the environment and the interaction between them. Such a system of scale-relativistic equations can be integrated in terms of a single multisystem Schrödinger equation (Nottale, 1995, 1996a), since the Schrödinger equation is the analog of the energy Hamilton–Jacobi equation. Such an equation has

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the fundamental property of non-separability, which may be highly relevant for the description of the relations between biological systems and their environment.

2.5. Gauge field theory

In the previous sections of the two companion papers, we have developed the subsequent steps of construction of the scale-relativity theory, namely:

(i) construction of the laws of scale at a given point in standard space–time as solutions of scale differential equations. These laws describe the explicit dependence of physical (and therefore also biological) quantities on scale variables (resolutions) that is expected as a direct consequence of abandoning the differentiability hypothesis. We have recovered the standard self-similar fractal laws, but also their symmetry breaking and some generalizations (scale dynamics);
(ii) construction of the laws of motion of such "fractally-structured points". We have shown that the most simple scale laws (constant fractal dimension $D_F = 2$) yield equations of motion (written under the form of geodesic equations) which can be integrated under the form of a Schrödinger-like equation. This equation can still be generalized in the cases of more complicated underlying scale-dynamical laws.

One may now go one step further, and consider a still more complete (and therefore more complicated and complex) situation. Up to now, the resolution variables ε were considered to be primary variables, of which the fractal coordinates were dependent, e.g., $\mathscr{L} = \mathscr{L}(\varepsilon)$. However, there is no reason for relativistic fractal structures as those considered here (which considerably differ from fractal objects) to remain rigid under displacements in space and time. In other words, we are naturally led to introduce a more general manifestation of a fractal and non-differentiable relativistic geometry, in which the resolutions themselves become a function of the positions and instants, namely, $\varepsilon = \varepsilon(x, y, z, t)$.

As we shall now briefly discuss, this generalized scale relativity manifests itself in terms of the appearance of gauge fields (of the electromagnetic type, but also of the strong and weak interaction type), which therefore receive a geometric interpretation in this framework (Nottale, 1994a; Nottale et al., 2008). The electromagnetic, weak and strong fields are thus interpreted as manifestations of the fractality of space–time, in analogy with the interpretation of the gravitational field as a manifestation of its curved geometry in Einstein's general relativity of motion.

Let us give a hint of how fields emerge in this framework.

Because, according to the principle of scale relativity, the scale space is fundamentally non-absolute, the scale of a structure internal to the fractal geodesics which describe, e.g., an electron, is expected to change during a displacement in space–time of that electron.

Therefore, we expect, under a displacement in space and time, the appearance of a resolution change due to the fractal geometry, which reads

$$\frac{\delta\varepsilon}{\varepsilon} = -\frac{1}{q}\sum_{\mu=1}^{4} A_\mu \, \mathrm{d}x_\mu. \tag{65}$$

where $\mathrm{d}x_\mu = t, x, y, z$ for $\mu = 0-3$. Because the elementary displacement in space and time is a four-vector, we are led to introduce another four-vector, A_μ, in order to describe the change of the resolution (assumed to be scalar at this first level of the analysis). As we shall now see, this "field" A_μ can actually be identified with an electromagnetic potential.

Let us indeed consider a second internal structure of the fractal geodesics that lies at a scale $\varepsilon'(x, y, z, t) = \rho(x, y, z, t) \times \varepsilon(x, y, z, t)$ relatively to the first one ε. Eq. (65) becomes for this new structure

$$\frac{\delta\varepsilon'}{\varepsilon'} = -\frac{1}{q}\sum_{\mu=1}^{4} A'_\mu \, \mathrm{d}x_\mu. \tag{66}$$

Now the physical effect of the potential A_μ should apply to the electron in its whole, and it should therefore be independent of the particular internal structure that we have followed. The new value of the potential can be

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calculated in function of the previous one. One finds

$$A'_\mu = A_\mu + q \frac{\partial}{\partial x_\mu} \ln \rho(x, y, z, t). \tag{67}$$

In other words, the physics should be unaffected by the addition to the potential A_μ of the gradient of any arbitrary function of space and time. This statement is exactly that of gauge invariance of the electromagnetic field, which has been discovered since the 19th century but without any understanding from first principles. In the new framework, it is obtained as a result and understood as a direct manifestation of the relativity of scales.

Other fundamental features of gauge theories can be constructed by the same method (Nottale, 1994a, 1996a; Nottale et al., 2008). In particular the charges themselves can be built as conservative quantities that emerge from the symmetries of the scale space (according, once again, to Noether's theorem), and the fundamental equations of these fields, classical and quantum, can be derived as geodesic equations written in terms of scale-covariant derivatives (Nottale, 1994a; Nottale et al., 2008).

Finally, this gauge field theory can be generalized to a more complex description of the scale variables, in terms (i) of four separated resolutions for the four coordinates instead of only one global dilation; (ii) and more generally, of a covariance matrix. It would describe an ellipsoid of resolution (similar to the ellipsoid of error used in data analysis), in which the correlations between the resolutions on different axes are taken into account.

This generalization allows one to obtain a geometric description of non-Abelian gauge fields as a manifestation of a general fractal space–time (Nottale et al., 2008).

Some of the properties of Abelian and non-Abelian gauge fields such as confinement (quarks cannot be separated by more than the nucleon diameter) or asymptotic freedom (the force decreases toward the small scales, so that the particles are free at relatively small scales and tied at large scales) could be particularly interesting after their reformulation in a biological context in terms of biological charges and biological fields.

3. Quantum-type mechanics in scale space

3.1. Motivation

Let us now consider another tentative development of the scale relativity theory founded on abandoning the hypothesis of differentiability of space–time coordinates. We reached the conclusion that the problem of dealing with non-derivable coordinates could be circumvented by replacing them by fractal functions of the resolutions. These functions are defined in a space of resolutions, or scale space.

The advantage of this approach is that it has displaced the problem of non-differentiability to infinity in the scale space (i.e., $\ln(\lambda/\varepsilon) \to \infty$), since, in terms of the new fractal tool $f(x, \varepsilon)$, only the limiting function $f(x, 0)$ is non-differentiable while $f(x, \varepsilon)$ can be derived in function of x and of ε when the resolution interval is non-zero.

In other words, everything becomes differentiable in terms of a double differential calculus (in space–time and in scale space). In such a framework, standard physics should be completed by scale laws allowing determination of the physically relevant functions of resolution. We have suggested that these fundamental scale laws be written in terms of differential equations, which amounts to define a differential fractal "generator". Then we have found that the simplest possible scale laws that are consistent (i) with the principle of scale relativity and (ii) with the standard laws of motion, lead to a quantum-like mechanics in space–time.

However, the choice to write the transformation laws of the scale space in terms of differential equations, even though it allows non-differentiability in standard space–time, implicitly assumes differentiability in the scale space. This is once again a mere hypothesis that can be abandoned in its turn.

We may therefore use in scale space the method that has been built for dealing with non-differentiability in space–time and explore a new level of structures that may be its manifestation (Nottale, 2004). As we shall now see, this results in scale laws that take quantum-like forms instead of classical ones.

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3.2. Schrödinger equation in scale space

Recall that, for the construction of classical scale differential equations, we have mainly considered two representations: (i) the logarithms of resolution are fundamental variables and (ii) the main new variable is the djinn τ (or scale time) and the resolutions are deduced as derivatives.

These two possibilities are also to be considered for the new present attempt to construct quantum scale laws. The first one consists of introducing a scale wave function $\psi(\ln \varepsilon(x,t), x, t)$. In the simplified case where it depends only on the time variable, $\psi = \psi(\ln \varepsilon(t), t)$, one may write a Schrödinger equation acting in scale space,

$$\mathcal{D}_\varepsilon^2 \frac{\partial^2 \psi}{(\partial \ln \varepsilon)^2} + \mathrm{i}\mathcal{D}_\varepsilon \frac{\partial \psi}{\partial t} - \frac{1}{2}\phi_\varepsilon \psi = 0. \tag{68}$$

We obtain this equation from the Schrödinger equation (39) by the replacement of x by $\ln \varepsilon$, then also of Δ by $\partial^2/(\partial \ln \varepsilon)^2$. The term ϕ_ε describes a possible scale potential acting on the inner fractal geometry of the system and deforming it with respect to the self-similar (scale-free) case, and $\mathcal{D}_\varepsilon$ is a self-organization parameter in scale space.

This equation is the quantum equivalent of the classical stationary wave equation giving rise to a log-periodic behaviour (see companion paper). It is also related to the scale relativity re-interpretation of gauge invariance, in which the resolutions become "fields" depending on space and time variables, so that the wave function becomes a function of $\ln \varepsilon$. However, only the phase is affected while the modulus of the scale wave function also depends on the resolution scale. This means that the solutions of such an equation give the probability of presence of a structure in the scale space, and that time-dependent solutions describe the propagation in the zoom dimension of quantum waves.

3.3. Complexergy

3.3.1. Third quantization

Let us consider now the second representation in which the djinn τ (or scale time that corresponds to a variable fractal dimension) has become the primary variable. (Recall that we have defined two approaches for the construction of scale laws: (i) the primary variables are the fractal length or coordinate (more generally, a fractal surface or volume) and the resolution, so that the fractal dimension is a derived quantity and (ii) the primary variables are the fractal length and the fractal dimension, become variable, and the resolution becomes a derived quantity in analogy with the velocity being the derivative of space with respect to time.)

In the case when the scale space is assumed to be differentiable, we have seen in the companion paper that the action principle can also be applied in it, and that this leads to write the equations of scale dependence in terms of scale Euler–Lagrange equations. In a simplified but fairly general case, they take (in terms of the djinn τ, which is the scale exponent become variable) the form of Newton's equation of dynamics, but now in scale space, namely, when the force derives from a potential,

$$\frac{\mathrm{d}^2 \ln \mathcal{L}}{\mathrm{d}\tau^2} = -\frac{\partial \Phi_S}{\partial \ln \mathcal{L}}. \tag{69}$$

Let us now abandon the hypothesis of differentiability of the scale space, while keeping its continuity. Since the scale space is now itself fractal, the various elements of the scale-relativistic description can be used again in this case, namely:

(i) infinity of trajectories, leading to introduce a scale velocity *field* $\ln(\lambda/\varepsilon) = \mathbb{V}(\ln \mathcal{L}(\tau), \tau)$, instead of the previously used deterministic scale velocities;

(ii) decomposition of d ln $\mathcal{L}$ in terms of a classical part and a fractal part, $\mathrm{d} \ln \mathcal{L} = \mathrm{d}x_S + \mathrm{d}\xi_S$, such that $\langle \mathrm{d}\xi_S^2 \rangle = 2\mathcal{D}_S \tau$, and two-valuedness because of the symmetry breaking of the reflection invariance under the exchange $(\mathrm{d}\tau \leftrightarrow -\mathrm{d}\tau)$;

(iv) introduction of a complex scale velocity field $\tilde{\mathcal{V}}$ based on this two-valuedness;

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(v) construction of a new total covariant derivative with respect to the djinn τ, which reads

$$\frac{\widehat{\mathrm{d}}}{\mathrm{d}\tau} = \frac{\partial}{\partial\tau} + \tilde{\mathscr{V}}\frac{\partial}{\partial \ln \mathscr{L}} - \mathrm{i}\mathscr{D}_s\frac{\partial^2}{(\partial \ln \mathscr{L})^2}. \tag{70}$$

It is constructed from Eq. (17) by the replacement of t by τ, of $\mathscr{V}$ by $\tilde{\mathscr{V}}$ and of x by $\ln \mathscr{L}$.

(vi) introduction of a wave function as a re-expression of the action (which is now complex), $\Psi_s(\ln \mathscr{L}, \tau) = \exp(\mathrm{i}\mathscr{S}_s/2\mathscr{D}_s)$;

(vii) transformation and integration of the above Newtonian scale dynamics equation under the form of a Schrödinger equation (cf. Eq. (39)) now acting on scale variables,

$$\mathscr{D}_s^2\frac{\partial^2\Psi_s}{(\partial \ln \mathscr{L})^2} + \mathrm{i}\mathscr{D}_s\frac{\partial\Psi_s}{\partial\tau} - \frac{1}{2}\Phi_s\Psi_s = 0. \tag{71}$$

This is therefore a kind of "third quantization" (Nottale, 2004), which is tentatively added to the first quantization of fermions and to the second quantization of fields (bosons).

3.3.2. A new conservative quantity

In order to understand the meaning of this new Schrödinger equation, let us review the various levels of evolution of the concept of physical fractals adapted to a geometric description of a non-differentiable space–time.

The first level in the definition of fractals is Mandelbrot's (1975, 1982) concept of fractal objects.

The second level has consisted of moving from the concept of fractal objects to scale-relativistic fractals. Namely, the scales at which the fractal structures appear are no longer defined in an absolute way: only scale ratios do have a physical meaning, not absolute scales.

The third level, that is achieved in the new interpretation of gauge transformations performed in scale relativity, considers fractal structures (still defined in a relative way) that are no longer static. Namely, the scale ratios between structures become a field that may vary from place to place and with time (see Section 2.5).

The final level (in the present state of the theory) is given by the solutions of the above scale-Schrödinger equation. The Fourier transform of these solutions will provide probability amplitudes for the possible values of the logarithms of scale ratios, $\Psi_s(\ln\rho)$. Then $|\Psi_s|^2(\ln\rho)$ gives the probability density of these values. Depending on the scale field and on the boundary conditions (in scale space), peaks of probability density will be obtained, this meaning that some specific scale ratios become more probable than others. Therefore, such solutions now describe quantum probabilistic fractal structures. The statement about these fractals is no longer that they contain given structures at some (relative) scales, but that there is a given probability for two structures to be related by a given scale ratio.

With regard to the solutions of the scale-Schrödinger equation, they provide probability densities for the position on the fractal coordinate (or fractal length) $\ln \mathscr{L}$. This means that, instead of having a unique and determined $\mathscr{L}(\ln\varepsilon)$ dependence (e.g., the length of the Britanny coast), an infinite family of possible behaviours is defined, which self-organize in such a way that some values of $\ln \mathscr{L}$ become more probable than others.

A new important quantity appears in this last representation. It is the conservative quantity which, according to Noether's theorem, must emerge from the uniformity of the new djinn variable (Nottale, 1993). It is defined in analogy with the definition of energy from the time symmetry in classical mechanics, in terms of the scale-Lagrange function $\tilde{L}$ and of the resolution $\mathbb{V} = \ln(\lambda/\varepsilon)$, as

$$\mathbb{E} = \mathbb{V}\frac{\partial\tilde{L}}{\partial\mathbb{V}} - \tilde{L}. \tag{72}$$

This new fundamental prime integral is therefore a "scale energy", i.e., it is the equivalent for scale of what energy is for motion. It has first been introduced in Nottale (1992, 1993), and it has been named "complexergy" in Nottale (2004).

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A more complete understanding of the meaning of this new description can be reached by considering an explicit example, e.g., the case of a scale harmonic oscillator potential well. Recall that such a potential (in the repulsive case) has already been considered as an example of new scale-dynamical laws (see companion paper) and has yielded a confinement-like behaviour. We shall now consider the quantum version of the effect of such a scale force (now in the attractive case). The stationary Schrödinger equation reads in this case:

$$2\mathcal{D}_s^2 \frac{\partial^2 \Psi_s}{(\partial \ln \mathcal{L})^2} + \left\{ \mathbb{E} - \frac{1}{2}\omega^2 (\ln \mathcal{L})^2 \right\} \Psi_s = 0. \tag{73}$$

The "stationarity" of this equation now means that it does no longer depend on the djinn (or scale time) τ. The behaviour of its solutions strongly depends on the complexergy $\mathbb{E}$, which can take only quantized values.

As we shall now see, the behaviour of the above equation suggests an interpretation for this conservative quantity and allows one to link it to the complexity of the system under consideration. It is the reason why we have suggested to call this new fundamental quantity complexergy (and also because it is linked to the djinn τ in the same way as energy is linked to the time, as conjugate quantities).

Indeed, let us consider the momentum solutions of the above scale-Schrödinger equation, i.e., scale momentum wave functions $a[\ln(\lambda/\varepsilon)]$. Recall that the main variable is now $\ln \mathcal{L}$ and that the scale momentum is the resolution, $\ln \rho = \ln(\lambda/\varepsilon) = \mathrm{d} \ln \mathcal{L}/\mathrm{d}\tau$ (since we take here a scale mass $\mu = 1$). The squared modulus of the wave function yields the probability density of the possible values of resolution ratios Fig. 8.

The complexergy is quantized as a consequence of the presence of the harmonic oscillator field. The various solutions (plotted in Fig. 9 as concerns the three first levels) depends on the quantized values of the complexergy, that read, in terms of the quantum number n

$$\mathbb{E}_n = 2\mathcal{D}_s \omega \left(n + \tfrac{1}{2}\right), \tag{74}$$

where ω is the frequency. As can be seen in Fig. 9, the solution of minimal complexergy shows a unique peak in the probability distribution of the $\ln(\lambda/\varepsilon)$ values. This can be interpreted as describing a system characterized by a single, more probable relative scale. Now, when the complexergy increases, the number of probability peaks $(n+1)$ increases. Since these peaks are nearly regularly distributed in terms of $\ln \varepsilon$ (i.e., probabilistic log-periodicity), they are distributed in scale (ε) as powers of a given unitary ratio a^n, so that they can be interpreted as describing a system characterized by a hierarchy of entangled levels of organization. A remarkable feature of this self-organization process is that the minimal complexergy, which yields the simplest structure with only one hierarchy level, is not vanishing: this vacuum complexergy is the scale analog of the vacuum energy of standard quantum mechanics.

Such a hierarchy of organization levels is one of the criterions that define complexity. Therefore increasing complexergy corresponds to increasing complexity, which is one of the justifications for the chosen name for the new conservative quantity.

More generally, one can remark that the djinn is universally limited from below ($\tau > 0$), which implies that the complexergy is universally quantized, and that we expect the existence of discretized levels of hierarchy of organization in nature (as actually observed) instead of a continuous hierarchy.

In the above situation, the structuring comes from the field, namely, an attractive harmonic oscillator potential in scale space. But this kind of behaviour is more general in such a quantum-type theory, as can be seen from another example. Assume that $\ln \mathcal{L}$ is limited at lower and upper scales and that the system is free (no applied force). The solution of the Schrödinger equation is in this case a log-periodic law of probability that takes a sinusoidal form (see Fig. 8)

$$P = a \sin^2(b \ln \mathcal{L}). \tag{75}$$

A similar result can also be directly obtained from the Schrödinger equation (68) written in terms of the resolutions $\ln \varepsilon$ (considered in this case as main variables). While the corresponding classical equation would describe, e.g., a continuous evolution with time of the characteristic size of a system, the quantum approach yields a punctuated evolution.

Indeed, if the system is led to increase its energy with time, as is the case of many living systems, its evolution is predicted to proceed from one entangled hierarchy of organization to a more complicated

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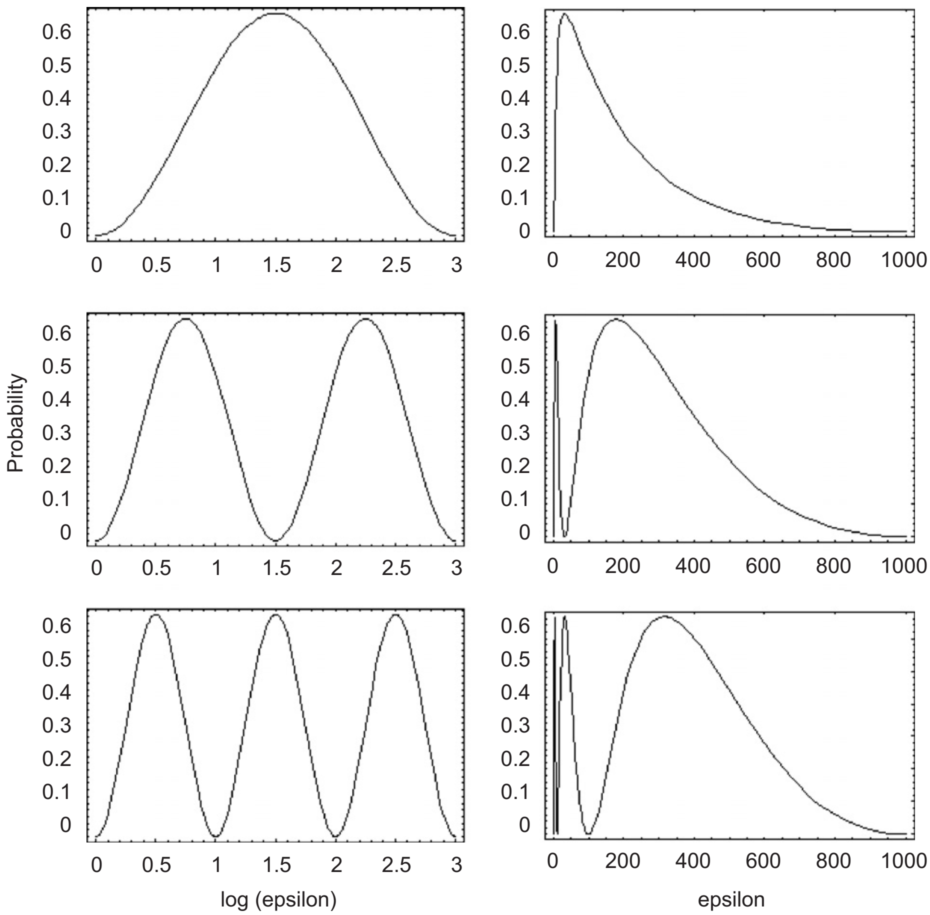

Fig. 8. Log-periodic solutions of the free Schrödinger equation written in terms of resolutions $\ln \varepsilon$. The left figures gives, for $n = 1-3$, the probability density $P(\ln \varepsilon)$ in function of $\ln \varepsilon$, and the right figures the corresponding probabilities in function of ε, in order to show the scale hierarchy of the probability peaks.

one by punctuated jumps rather than by a continuous process, since only the quantized states are in equilibrium.

Therefore these solutions may yield an explanation for the discrete scale invariance describable by log-periodic laws observed in many biological systems (see companion paper).

3.4. Application to biological systems

The various new concepts introduced in this section (Schrödinger equation in scale space, complexergy) are particularly adapted to applications in biology. Indeed, we have recalled in the companion paper that several lineages of the tree of life, including the first events of species evolution, can be described in terms of a log-periodic acceleration or deceleration law (Chaline et al., 1999; Nottale et al., 2000b; Nottale et al., 2002). The same is true of human development (embryogenesis and child development, Cash et al., 2002). These models have been voluntarily limited (as a first methodological step) to an analysis of only the chronology of events, independently of the nature of the major evolutionary leaps. But we have now at our disposal a tool that allows us to reconsider the question.

We have indeed suggested that life evolution proceeds in terms of increasing quantized complexergy and/or energy (Nottale, 2004). This would account for the existence of punctuated evolution (Gould and Eldredge, 1977), and for the log-periodic behaviour of the leap dates recalled in the first companion paper, which can

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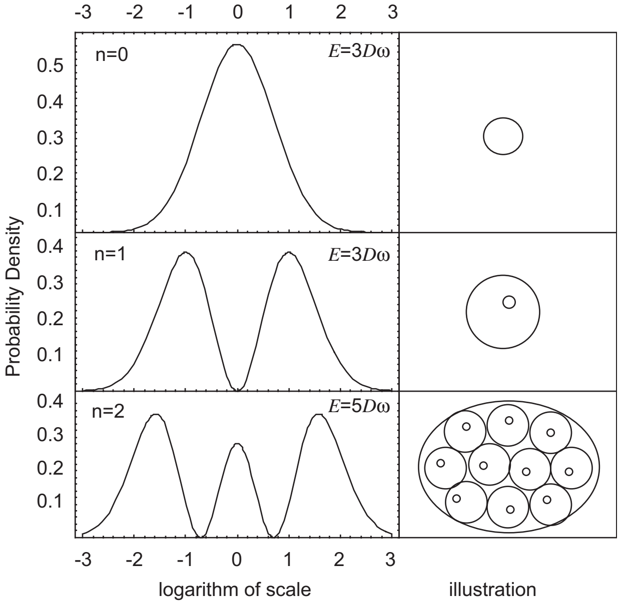

Fig. 9. Solutions of increasing complexergy of the scale-Schrödinger equation for a harmonic oscillator scale potential. They provide a probability density to show a structure at a (relative) scale. These solutions can be interpreted as describing systems characterized by an increasing number of hierarchical levels, as illustrated in the right-hand side of the figure. For example, living systems such as prokaryotes, eukaryotes and simple multicellular organisms have, respectively, one characteristic scale (cell size), two characteristic scales (nucleus and cell) and three characteristic scales (nucleus, cell and organism). One obtains a similar result by solving a Schrödinger equation in resolution space under box-limiting conditions (see Fig. 8).

now be interpreted in terms of probability density of the events. Moreover, one may contemplate the possibility of an understanding of the nature of the events, even though in a rough way as a first step.

Indeed, let us consider the free Schrödinger equation in scale space. Its solutions are determined by the limiting conditions, in particular by the minimal and maximal possible scales. One expects the formation of a structure at the fundamental level (lowest complexergy) characterized by one length scale (Fig. 9). Moreover, the most probable value for this scale of formation is predicted to be the middle of the logarithmic scale space.

Let us evaluate the value of this length scale. One can first consider the full physical scale space (see Fig. 2 of companion paper, right). The universal boundary conditions are the Planck-length l_P in the microscopic domain and the cosmic scale $\mathbb{L} = \Lambda^{-1/2}$ given by the cosmological constant Λ in the macroscopic domain (see Nottale, 1993, 1996a, 2003) for an interpretation of these scales as limiting, unpassable minimal and maximal scales, invariant under dilations. From the predicted value of the cosmological constant (Nottale 1993, 1996a) which is now supported by observational results (Spergel et al., 2003), one finds $\mathbb{L}/l_{lP} = (5.300 \pm 0.001) \times 10^{60}$, so that the mid scale is at $2.3 \times 10^{30}\, l_P \approx 40\,\mu\text{m}$, which is indeed a typical scale of living systems (eukaryotic cells).

One can also consider only a subspace of the full scale space that contains the biological scales only (left of Fig. 2 of companion paper). The minimal scale concerning biological processes can be taken to be the atomic scale, given by the Bohr radius at 0.5Å, while the maximal length scale of animals (whales) is of the order of 30 m, or $(5\,\text{m})^3$ in volume if one wants to account for their anisotropy. Using these biotic or prebiotic boundaries, one finds a geometric mean of these extreme scales that is once again of the same order ($15-40\,\mu\text{m}$). It is remarkable that the biological scale subspace and the full physical scale space be centred one on the other.

Moreover, this μm scale is indeed a typical scale of living cells. For example, the size of bacteria is typically one to several μm, while red and white blood cells range between 5 and 30 μm in size. Moreover, the prokaryotic cells, which are the first living systems appeared more than 3 Gyrs ago, are precisely characterized by this typical size range and by having only one hierarchy level (no nucleus).

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In this framework, a further increase of complexergy can occur only in a quantized way. The second level describes a system with two levels of organization, in agreement with the second step of evolution leading to eukaryotes about 1.7 Gyrs ago. One expects (in this very simplified model) that the scale of nuclei be smaller in general than the scale of prokaryotes, itself smaller than the scale of eukaryotes: this is indeed what is observed.

The next expected major evolutionary leap is a three-level organization, in agreement with the appearance of multicellular forms (animals, plants and fungi) about 1 Gyr ago. It is also predicted that the multicellular nucleated stage can be built only from eukaryotes, in agreement with what is observed. More generally, the evolution of living organisms proceeds on the basis of previous evolution (systems with memory).

The following major leaps correspond to the emergence of more complicated structures, then of new functions (supporting structures such as exoskeletons, tetrapody, homeothermy, viviparity), but they are still characterized by fundamental changes in the number of organization levels. The above model (based on spherical symmetry) is clearly too simple to account for the events that follow the first spherical exoskeletons, but it can be naturally extended to the description of anisotropic structures. The theoretical biology approach outlined here is still in its infancy: future attempts of description using the scale-relativity methods will have the possibility to take into account more complicated symmetries, boundary conditions and constraints, so that such a new field of research (which may become a predictive theoretical biology) seems to be wide open to investigation.

The application of this model to embryogenesis has also yielded encouraging results, since we expect from it both a temporal and a spatial hierarchy of organization described by log-periodic laws, as actually observed (see Cash et al., 2002 and companion paper).

4. Discussion, conclusions and perspectives

4.1. Validated predictions of scale relativity theory in astrophysics

The theory of scale relativity has been able from its beginning (Nottale, 1989, 1992, 1993) to provide several new results in physics and astrophysics, as well as theoretical predictions of new effects. A list of such results and predictions has been given 10 years ago in Section 9 of Nottale (1996a). Most of these predictions have been subsequently verified by new observational and experimental data, and still new theoretical predictions have been more recently proposed (da Rocha and Nottale, 2003a; Nottale, 2003).

We shall not give here an exhaustive list, but simply provide the reader a hint of the kind of possible applications of the theory, particularly concerning the domain which is the closest to the questions addressed in biology, namely, the self-organized process of formation and evolution of structures in astrophysics.

In this realm the organizing field is essentially the gravitational field. From the very beginning of the construction and development of a macroscopic quantum-like theory (Nottale, 2003), it appeared as quite natural to describe gravitational structure formation by a generalized Schrödinger equation based on a self-organization constant (instead of the Planck constant of standard quantum mechanics). The first application (Nottale, 1993, Section 7.2) of this approach (which actually is a form of what is now called migration theories, which account for a coupling between protoplanets and the protoplanetary disk), to the formation of planetary systems, has yielded a theoretical prediction of the expected probability density distribution of planets which compared very well with their observed distribution in our own solar system.

Moreover, one year before the discovery by Mayor and Queloz of the first exoplanet around a solar-type star, one of us could write (Nottale, 1994b): ‘‘We can expect [...] other systems to be discovered in the forthcoming years, and new informations to be obtained about the very distant solar system (Kuiper's belt, Oort cometary cloud...). In this regard our theory is a falsifiable one, since it makes definite predictions about such observations of the near future: observables such as the distribution of eccentricities, mass, angular momentum, the preferred positions of planets and asteroids, or possibly the ratio of distance of the largest gazeous planet and the largest telluric one, are expected in our framework to be universal structures shared by any planetary system.’’

This prediction has subsequently been supported by new observational data, in particular by the discovery of extrasolar planetary systems after 1995. Indeed, the theory has been able to predict in a quantitative way a

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large number of new effects in the domain of gravitational structures (da Rocha and Nottale, 2003), and these predictions have been successfully checked in various systems on a large range of scales and in terms of a common gravitational coupling constant (or one of its multiples or submultiples) (Nottale, 1996b). New structures have been theoretically predicted, then checked by the observational data in a statistically significant way, for our solar system, including distances of planets (Nottale, 1993; Nottale et al., 1997) and satellites (Hermann et al., 1998), sungrazer comet perihelions (da Rocha and Nottale, 2003a), obliquities and inclinations of planets and satellites (Nottale, 1998b), density peaks of infrared intramercurial dust (da Rocha and Nottale, 2003a), distribution of objects in the Kuiper belt (da Rocha and Nottale, 2003a), period of the Sun cycle (Nottale, 2004); then for extrasolar planetary systems, including exoplanets semi-major axes (Nottale, 1996b) and eccentricities (da Rocha and Nottale, 2003a), planets around pulsars, for which a high precision is reached (Nottale, 1996b); then for double stars (Nottale and Schumacher, 1998), planetary nebulae (da Rocha and Nottale, 2003a, b), binary galaxies (Nottale, 1996a; da Rocha and Nottale, 2003a), our local group of galaxies (da Rocha and Nottale, 2003a), clusters of galaxies and large scale structures of the universe (Nottale and Schumacher, 1998; da Rocha and Nottale, 2003a).

In most of these cases, the theory has been able both to account in a quantitative way for unexplained structures and to predict the existence of new, not yet observed structurings that are expected to be discovered in the future by the new generation of large telescopes (see in particular da Rocha and Nottale, 2003a).

Let us finally remark that a Schrödinger form has also been obtained for several other fundamental equations of physics, since the general method described above can be applied to any physical situation where the three basic conditions (namely, infinity of trajectories, fractal dimension 2 of trajectories, breaking of differential time reflexion invariance) are achieved in an exact or in an approximative way. We cannot enter here into the detail of these generalizations, so we refer the interested reader to Nottale (1997).

4.2. *Founding elements of an integrative theory of life on first principles*

Throughout these two papers, we have provided indications that the scale relativity framework offers the opportunity to find on common first principles elements of the evolutionary, developmental, biochemical and cellular theories that were previously inferred from observations as axiomatic principles:

(i) the structure of the tree of life with its accelerating and decelerating temporal paths between major events in different lineages can be described by a log-periodic generalization of scale invariance (Section 3.4.1 of first paper);
(ii) the formation of cellular structures may be related to the property of confinement induced by scale dynamics resulting in departures from scale invariance (Section 3.4.2 of first paper);
(iii) the increase in the number of hierarchical levels during evolution, starting with the formation of cellular structures, can be described as a transition from the fundamental state to excited levels of a new quantized entity, complexergy, in the framework of a quantum description of scale space (section 3.4 of this paper); and
(iv) cell duplication and morphogenesis may be the consequence of the discrete symmetry breaking property of quantum-like dynamics in fractal space, resulting in a quantum-type macroscopic behaviour (Section 2.4.1 of this paper).

The initial models outlined are obviously too simple to account for the full complexity of biological system behaviours, which are the result of the interplay of functional and regulatory networks modulated by component properties and external environmental influences. Their merit and usefulness is to provide examples of the ability of the macroscopic Schrödinger theory to provide generic models of biological functions (morphogenesis, duplication, confinement by walls, multiscale hierarchy of organization, etc.) which correspond to underlying geometric constraints in biological systems with no or poor analogs in classical physics. In the step-by-step approach of the scale relativity theory framework, these simple generic functions are first described separately, and then can be integrated in an attempt to account for biological system behaviours.

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Actually, the given examples were mainly intended to show that, in the scale relativity framework, one expects the existence of fundamental relations between these functions and the basic conservatives quantities (e.g., duplication linked to energy jumps, change of number of hierarchical levels linked to complexergy jumps, morphologies linked to other quantum numbers, etc.), and that these conservative quantities are quantized. We intend to put these theoretical predictions, among others, to the test in actual experiments, and we hope that, since all of them are issued from the same theoretical framework, these various functions could be integrated under a common model in the next step of the theoretical program.

It is already apparent from this initial list that the scale relativity framework reflects on the periodic and oscillatory behaviours that have been uncovered in living systems and the subject of a number of advanced studies over the past several decades (see e.g., Goodwin, 1965; Goldbeter and Decroly, 1983; Goldbeter and Martiel, 1987; Goldbeter, 1996, 2002; Françoise et al., 2003; Françoise, 2004; Garcia-Ojalvo et al., 2004).

Moreover, this intrinsic symmetry breaking property may be the basis for the twin-coupled complementary processes of self-organization of complex biological systems, possibly induced by a quantum-type anti-diffusion process, generating neguentropy, and the reaction–diffusion process underlying biochemical reactions, generating disorganization and entropy (Section 2.3.6 of this paper). This interpretation has the merit of reconciling the apparently contradictory proposals of classical thermodynamics and dynamic system theories put forward and hotly debated by communities of physicists and chemists (Anderson, 1995; Stein, 1980), mathematicians and computer scientists (Wolfram, 1985; Bak et al., 1988; Strogatz, 1994; Alligood et al., 1997; Blanchard et al., 2000; Bailly and Longo, 2003; Cessac et al., 2004; Aubin and Lesne, 2005) and biologists (Varela, 1995; Thomas and Kaufman 2001a, b, Snoep et al., 2006). It also provides a foundation for our first conjecture that the self-organization of complex biological systems is a result of a conjunction of a relatively stable organization in a framework of multiple low intensity fluctuations operating in biological space–time (Auffray et al., 2003a). As a matter of fact, stochastic description of such fluctuations is at the heart of scale relativity theory: combined with irreversibility, they lead to transformation of the equations of classical motion into quantum-type equations having the same form as the Schrödinger equation.

4.3. Defining biological space– time, biological fields and charges

However, in order to prevent any misconception, it is important to be more specific about what we mean here by biological space–time. It must be clear from now on that this does not concern the same level of space–time description as in the gravitation theory of general relativity, or as in the application of scale relativity to the standard quantum mechanics of atomic, nuclear and elementary particle physics. In these two cases, it is the very definition of what is space and time which is affected in a fundamental way, and this is confirmed by actual measurements of time and space intervals which lead to results irreducible to the conceptions of classical pre-relativistic physics. We do not expect the same to occur in a biological medium: namely, physical rules and clocks are probably not deformed at scales typical of living systems, although biological rules and clocks may behave differently.

So the suggested intervention of the concept of space here has another meaning: it is rather an effective concept aimed at describing the medium itself and its interrelations with its objects.

This point can be better understood by a well-known example in physics. The electroweak theory of Glashow, Salam and Weinberg (see, e.g., Aitchison, 1982) has been constructed by using an analogy with superconductivity. A superconducting medium is characterized by the existence in it of electron pairs, whose condensate forms a new ground state that changes the effective mass and charges of the other electrons of the material. Similarly, in the electroweak theory one postulates the universal existence of a bosonic field, the so-called Higgs field, that changes the vacuum energy and therefore allows the emergence of masses for the weak gauge bosons W and Z, which would otherwise be massless. A third level of description of the electroweak interaction amounts to consider the fields themselves (both the electroweak and Higgs fields) as a very manifestation of a non-differentiable space–time geometry (Nottale, 2000; Nottale et al., 2008).

One may reverse the argument of this material-field-geometry analogy and suggest that a space–time theory can be used for the description of a medium, of its inner objects and of their relations, in particular in biological systems. In such an approach, the properties of the objects, which are contained in the space, are linked and changed, if not defined, by those of the space.

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This suggests the intriguing possibility of defining biological fields and charges by applying to biological parameters the same progressive construction that was used for the exploration of scale space, thus providing a means to formalize the notion of biological space–time and justify our second conjecture that biological systems operate in a variable number of dimensions, (Auffray et al., 2003a) opening the way to a sort of "scale-relativistic biology".

It should be stressed, however, that much remains to be done before these opened avenues materialize into novel practical applications, a necessary and essential step to eventually validate the relevance of the scale relativity conceptual framework and the usefulness of the mathematical formalisms developed to implement it.

4.4. Extending the classical framework for multiscale integration

The scale relativity framework offers the possibility to find on first principles approaches which developed on empirical or modelling bases. Through extension and generalization, it provides the opportunity to explain phenomena and operating principles that could not be fully understood in the classical physical framework, building on previous knowledge.

General relativity theory did not dismiss the classical Newtonian theory and its wide range of practical applications in everyday's life, but extended it to provide the opportunity for novel understanding on astronomical scales together with the development of novel devices such as for example the GPS for precise global positioning. We expect the situation to be the same with a scale-relativistic biology which will not replace but integrate and extend the potential of current biology in its multiple flavours, and potentially shed new light on a variety of biological phenomena.

For example, the quantum-type fluid mechanics form of the motion equation described in Section 2.3.5 of this paper could, if validated experimentally, be used to further develop and integrate models of circulation of fluids, energy, and possibly information, in biological systems. It could help derive from first principles established laws such as Starling law for bioenergetics which states that "within physiological limits, the force of contraction is directly proportional to the initial length of the muscle fibre" (reviewed in Saks et al., 2006).

This intrinsic iterative and integrative character should prove extremely useful to sustain the development of integrative systems biology by resolving in a very fundamental way the problems of multiscale integration. Indeed, as described in the first paper, the concept of scale space integrates the various scales, their specific laws, their transformations, transitions and limits (which appear as "objects" contained in the scale space) in a unique limitless thoughtframe (the space which is the "container", defined as the interrelational level of description between the objects) that may transcend even multiscale approaches.

It is of course not our intention to dismiss the value of the existing experimental tools and models which have demonstrated and well-established properties and performances to deal with certain classes of biological questions and situations. The Physiome Project is a prime example of how far the current tools have enabled progress and of the limitations faced (Noble, 2002a, b; Hunter and Borg, 2003; Hunter and Nielsen, 2005). As pointed out in the first paper, it is widely recognized that the mathematical and computational formalisms used for numerical simulation (see e.g., Onsager, 1931; Gillespie, 1976; Britton, 1986; Keener and Sneyd, 1998; Perko, 1991; Longo, 2003) or network modelling (e.g., Segre et al., 2002; Kholodenko et al., 2002; Demongeot et al., 2003; Burgard et al., 2004; Kobayashi et al., 2004) have a long history of independent development but are often incompatible in their basic principles. Hence, despite the efforts put on integration of multiple data types (e.g., Hwang et al., 2005a, b) and on the recent development of model exchange platforms such as BioModels (2006), multiscale integration remains the grand challenge of integrative systems biology (see references of Section 1.2 in the first paper).

The scale relativity framework also suggests a path for the design of macroscopic quantum-type experiments and devices operating on scale-relativistic biological principles, as discussed below.

4.5. Design of macroscopic quantum-type experiments and devices

Let us finally give another example of application of the scale relativity framework which may also be particularly relevant to biology.

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In the above application to astrophysics, the effects of fractality and non-differentiability, and of their consequence as macroscopic quantum-type self-organization have been searched for in natural systems. There is, however, another way to study experimentally the theoretical predictions of the scale relativity framework. It consists, instead of searching for a spontaneous implementation in natural systems of the process it describes (namely, of the emergence from the fractal geometry of space of a macroscopic quantum-type mechanics), of constructing this process in an experimental and controlled way.

In this context, a new class of macroscopic quantum-like laboratory experiments has recently been proposed (Nottale, 2008; Nottale and Lehner, 2006). It consists of applying, through a real time retro-active loop, a generalized quantum potential on a classical system. Indeed, we have shown that the system of equations (Euler equation and continuity equation) that describes a fluid in irrotational motion subjected to a generalized quantum potential, is equivalent to a generalized Schrödinger equation (Nottale, 1993, 1997; Nottale et al., 2000a, b). Moreover, it can be shown (Nottale, unpublished) that, even in the case of the presence of vorticity (rotational motion), it is also possible to obtain a Schrödinger-like equation (of the electromagnetic type) for a large class of systems. In this derivation, the quantum potential is no longer founded on the quantum Planck's constant $\hbar$, but on a new constant $\mathcal{D}$ which can take any macroscopic value and which gives back standard quantum mechanics in the special case $\mathcal{D} = \hbar/2m$. Therefore such a system is expected to exhibit some quantum-like macroscopic properties. Moreover, since the Euler and continuity system of equations can also be used as approximation in the description of many other types of physical systems (chaotic mechanical systems, n particle dynamics, etc.), it is a full class of new experimental quantum-like systems that can be proposed in such a framework.

Such experiments, provided they could be effectively implemented, would be in the same time laboratory experiments of physics, of astrophysics and of biology: for example, similar general conditions, such as a jet issued from a centre, correspond to a similar kind of solutions (outgoing spherical waves), and simultaneously lead to a representation of scattering of a particle on an atom (physics), ejection of outer shells from a star (planetary nebulae: astrophysics) or to the growth from a centre that leads to flower-like shapes (see Figs. 2 and 3: biology).

Such an approach may therefore lead to a new quantum-type, artificial life technology. Indeed, the retro-action loop that would be used in this experiment to apply the quantum potential is typical of biofeedback in biological systems. It involves:

(i) measurement of a physical quantity, such as a density, by a measurement apparatus; this first step of the loop is similar to the detection of an information by the sense organs;
(ii) calculation of the quantum potential and of the quantum force (by using a computer in the experiment) from the measured quantity; this second step is similar to a cognitive process;
(iii) application of the computed force on the system by actuators; this third step is similar to the interaction of a biological system with its environment through the use of action organs.

Moreover, the ability of the experimentalist to control the amplitude of the quantum potential in such an experiment or device, and even possibly to reverse it (see Section 2.3.6), would offer a unique occasion to study the transition from a quantum-type self-organization, to a classical causal-type organization (vanishing quantum potential) then to a diffusion-like disorganization (reversed quantum potential). Such devices could be viewed both as biologically inspired synthetic biology constructions having some of the properties usually attributed to living systems such as spontaneous duplication (when some conditions are fulfilled), hierarchy of organization, and confinement. They should play for integrative systems biology the same role as the successive different types of microscopes for the emergence and development of cellular and molecular biology, and as the increasingly powerful telescopes in physics and astrophysics (note in this regard that the proposed retro-active loop is reminiscent of adaptive optics in instrumental astronomy). Such devices, combining the analytical and systemic principles (Fig. 10), could be used for example in environment management and exploration, and as monitoring and reactive devices for piloting of biological systems in proximity of healthy state trajectories, with high diagnostic and therapeutic potential.

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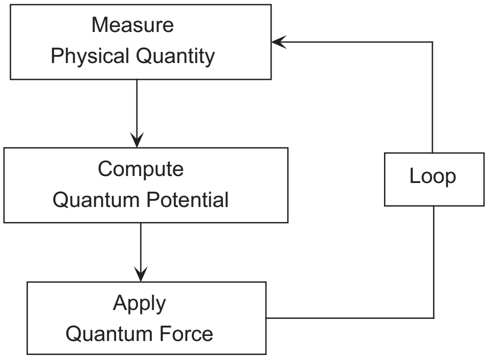

Fig. 10. The retro-active loop, similar to a biofeedback, which underlies the design of a new macroquantum experiment and technology, complying with the analytical and systemic precepts (see Fig.1 of first paper).

4.6. Applications in integrative systems biology

Macroscopic quantum-type experiments and devices could be instrumental in realizing the synthetic and engineering biology agenda (Kitano, 2002; Brent, 2004; Benner and Sismour, 2005; Endy, 2005; Andrianantoandro et al., 2006; Doyle and Stelling, 2006; Drubin et al., 2007; Tyo et al., 2007) and in meeting the challenges faced in drug and therapeutic development, diagnosis and treatment monitoring (Hood and Perlmutter, 2004; Butcher et al., 2004; Loging et al., 2007). Recent advances in experimental design and measurement technologies (see e.g., Ransohoff, 2002; Simon, 2003, 2005; Hood et al., 2004; Imbeaud and Auffray, 2005) have started to provide access to the low intensity signals and fluctuations that appear essential to drive the behaviour of biological systems, often negatively designated as "noise" (McAdams and Arkin, 1997; Hasty et al., 2000; Raser and O' Shea, 2005). They should be considered as the potential inputs of future quantum-type devices.

Indeed, we recently deciphered the cellular states of innate drug responses through a combination of high-precision transcriptome analyses, functional annotation and systemic modelling (Graudens et al., 2006) and found that largely overlapping sets of genes associated with a large number of small changes in expression account for both the underlying mechanism of tumour resistance and the ability to predict the outcome of chemotherapy in cancer patients (Auffray et al., unpublished results). We are currently actively exploring the fields of cancer and immunity (see e.g., Weinstein et al., 1997; Hanahan and Weinberg, 2000; Rosenwald et al., 2002; Padua et al., 2003; Charron, 2005; Dickinson and Charron, 2005); pulmonary and muscular diseases (e.g., Jafri et al., 2001; Guerrero et al., 2005), taking also advantage of progress in gene transfer and gene silencing technologies (e.g., Brun et al., 2003; Amar et al., 2006; Bemelmans et al., 2006) to develop the pilot projects in integrative systems biology of the SYSTEMOSCOPE Consortium (Auffray et al., 2003b) in the framework of scale relativity theory.

4.7. Consequences for transdisciplinary research, development and training in integrative systems biology

Throughout our journey at the interface of scale relativity theory and integrative systems biology, it has not escaped our attention that adoption of this extended framework has radical and far-reaching consequences for the organization of teaching, research and development, and a wide range of possible applications for energy and environment management, agriculture and health to mention only fields directly or indirectly related to biology.

The diversity of the natural sciences and the question of their interdependence have been addressed since the Greek philosophers. The current hierarchy dominated in the scientific curriculum and research organizations by mathematics and formal sciences, followed by physics, chemistry and biology and the related engineering disciplines is clearly inherited from the analytical reductionist framework which dominated the scene since Galileo and Descartes. The relevance of this hierarchy has been disputed repeatedly during the past two millennia, and predates the creation of universities. It was, for example, the subject of a famous *disputatio*

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between two prominent cathedral masters at the end of the 10th century, when Ottric from Magdeburg had accused Gerbert d'Aurillac to have subordinated physics to mathematics in his teaching curriculum (Trystram, 2000). Gerbert, one of the greatest minds of his time, had re-introduced the *quadrivium* (arithmetic, geometry, astronomy and music) in the teaching curriculum of the cathedral school in Reims, and is famous among other things for substituting the Arabic numbers of Indian origin to the Roman numbers, thus facilitating the development and spread of the calculus in Western science and society. When given the opportunity to defend his views publicly by the German Emperor Otto II in Ravenna in 981, he brilliantly dismissed Ottric's accusation and explained why there was no *a priori* reason for any of the natural sciences, including mathematics, to depend on another, in agreement with Aristotles' classification of the branches of the philosophy of nature. Impressed by the clarity of his arguments, Otto ruled in his favour, an episode which greatly contributed to the fame of Gerbert (in 999, Otto III chose Gerbert to become the pope of the Catholic Church under the name of Sylvester II).

A millennium later, the framework offered by the scale relativity theory opens the opportunity to "rethink research in order to understand life and improve health", the motto of the SYSTEMOSCOPE Consortium (Auffray et al., 2003b). We argued with others that implementation of this renewed vision of the research curriculum will be best achieved by the combination of a network of interactions between the different disciplines, greatly facilitated by communication through the Internet, and the creation of centres dedicated to the development of integrative systems biology and its applications for human health, where scientists of all origins will work together on joint long-term programmes. In this context, system engineering and computer sciences, including micro- and nanotechnologies and modelling and simulation will play a decisive role, and the importance of constant interactions with philosophy and history of sciences, law and ethics should not be underestimated. Cross-disciplinary training will be essential to equip individual scientists with a mutual understanding and a common language. This will enable small and large teams to leverage on their diverse range of expertise to establish more profound foundations for a theory of biology through the integration of the cell, biochemical and evolution-development theories based on first principles. It will stimulate the development of practical applications based on the design and control of the macroscopic quantum-type behaviour of physical and biological systems that is proposed here. We invite academic and industrial partners to join us in this endeavour, and to provide their insight, expertise and support for implementation of the SYSTEMOSCOPE Consortium research, development and training agenda, to support the emergence of a new philosophy of nature.

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Appendix A. Proof of fundamental remarkable identity

Let us prove the remarkable identity (33) that plays a central role in the derivation of the generalized Schrödinger equation (Nottale, 1993), namely,

$$\nabla\left(\frac{\Delta\psi}{\psi}\right) = 2(\nabla \ln\psi.\nabla)(\nabla \ln\psi) + \Delta(\nabla \ln\psi). \tag{76}$$

Knowing that ∇ and $\varDelta$ commute and that $\varDelta = \nabla^2$, we may write

$$\begin{aligned}\varDelta(\nabla \ln \psi) &= \nabla(\varDelta \ln \psi) = \nabla(\nabla.\nabla \ln \psi) = \nabla\left(\nabla.\frac{\nabla\psi}{\psi}\right)\\ &= \nabla\left\{\frac{\Delta\psi}{\psi} - \left(\frac{\nabla\psi}{\psi}\right)^2\right\} = \nabla\left(\frac{\Delta\psi}{\psi}\right) - \nabla\left(\frac{\nabla\psi}{\psi}\right)^2\\ &= \nabla\left(\frac{\Delta\psi}{\psi}\right) - 2\left(\frac{\nabla\psi}{\psi}\right).\nabla\left(\frac{\nabla\psi}{\psi}\right) = \nabla\left(\frac{\Delta\psi}{\psi}\right) - 2(\nabla \ln \psi.\nabla)(\nabla \ln \psi),\end{aligned} \tag{77}$$

and therefore we recover the above identity. Note that the passage from the first to the second line can be done, e.g., by decomposing the scalar product in each of its components on x, y, and z, namely,

$$\begin{aligned}\nabla.(\nabla\psi/\psi) &= \sum_{xyz} \frac{\partial}{\partial x}\frac{\partial\psi/\partial x}{\psi} = \sum \frac{\psi\partial^2\psi/\partial x^2 - (\partial\psi/\partial x)^2}{\psi^2}\\ &= \sum \frac{\partial^2\psi/\partial x^2}{\psi} - \sum \frac{(\partial\psi/\partial x)^2}{\psi^2} = \frac{\Delta\psi}{\psi} - \left(\frac{\nabla\psi}{\psi}\right)^2.\end{aligned} \tag{78}$$

The identity can still be easily generalized by replacing in it ψ by ψ^α. One obtains the more general identity of C.3.2.

Appendix B. From Schrödinger equation to Euler and continuity equations

By separating the real and imaginary parts of the Schrödinger equation and by using a mixed representation, (P, V) (instead of (V, U) in the geodesics form and (P, θ) in the Schrödinger form), one obtains fluid dynamics-like equations, i.e., a Euler equation and a continuity equation (this is a generalization of the Madelung transformation, but whose physical meaning is here set from the very beginning).

Let us recall explicitly this transformation. We first come back to the definition of the wave function by making explicit the probability and the phase, namely,

$$\psi = \sqrt{P} \times e^{iS/2m\mathscr{D}}. \tag{79}$$

By introducing this form of the wave function in the Schrödinger equation with an external scalar potential ϕ, we obtain

$$\left\{-\frac{\sqrt{P}}{2m}\left(\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + \Phi - 2m\mathscr{D}^2\frac{\Delta\sqrt{P}}{\sqrt{P}}\right) + i\frac{\mathscr{D}}{2\sqrt{P}}\left(\frac{\partial P}{\partial t} + \mathrm{div}\left(P\frac{\nabla S}{m}\right)\right)\right\}e^{iS/2m\mathscr{D}} = 0. \tag{80}$$

Now, the complex velocity $\mathscr{V} = V - iU$ being linked to the wave function by the relation $\mathscr{V} = -2i\mathscr{D}\nabla \ln \psi$, its real part is therefore given in terms of the phase by the relation (Nottale, 1993)

$$V = \frac{\nabla S}{m}. \tag{81}$$

Note that this fundamental relation is here derived from the initial description of the fluid of geodesics, while V is built from S itself in the standard Madelung transformation, so that its meaning is interpreted *a posteriori*. In the scale relativity/non-differentiable space–time approach, the velocity field and the probability density characterize from the beginning the bundle of geodesics.

By replacing in the above form of the Schrödinger equation $\nabla S/m$ by the real velocity field V, it reads

$$\left\{-\frac{\sqrt{P}}{2m}\left(\frac{\partial S}{\partial t} + \frac{1}{2}mV^2 + \Phi + Q\right) + i\frac{\mathscr{D}}{2\sqrt{P}}\left(\frac{\partial P}{\partial t} + \mathrm{div}(PV)\right)\right\}e^{iS/2m\mathscr{D}} = 0, \tag{82}$$

so that we finally obtain an Euler-type equation and a continuity equation, namely,

$$\left(\frac{\partial}{\partial t} + V \cdot \nabla\right)V = -\nabla\left(\frac{\Phi + Q}{m}\right), \tag{83}$$

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$$\frac{\partial P}{\partial t} + \text{div}(PV) = 0, \tag{84}$$

which includes an additional potential energy Q that writes

$$Q = -2m\mathscr{D}^2 \frac{\Delta\sqrt{P}}{\sqrt{P}}. \tag{85}$$

This scalar potential generalizes to a constant $\mathscr{D}$ (which may be different from $\hbar/2m$) the quantum potential obtained in the Madelung–Bohm transformation. But here the velocity field is not interpreted from the transformation itself, but is instead that which has been introduced from the very beginning as the geodesics velocity field. In the same way, the probability density is also easily interpreted in this framework as arising from the distribution of geodesics, as can be verified by numerical simulations (Hermann, 1997). Moreover, this transformation can be shown to be reversible for potential motion: namely, an irrotational classical fluid subjected to a quantum potential becomes describable by a Schrödinger equation (possibly of the non-linear type, see Nottale and Lehner, 2006).

Appendix C. Schrödinger process as anti-diffusion process

C.1. Fluid representation of diffusion processes

Let us consider a classical diffusion process (such as, e.g., a Brownian motion of small particles in a fluid). Such a process is described by the Fokker–Planck equation:

$$\frac{\partial P}{\partial t} + \text{div}(Pv) = \mathscr{D}\Delta P, \tag{86}$$

where $\mathscr{D}$ is the diffusion coefficient, P the probability density distribution of the particles and $v[x(t), t]$ is their mean velocity.

When there is no global motion of the diffusing fluid or particles ($v = 0$), the Fokker–Planck equation is reduced to the usual diffusion equation for the probability P:

$$\frac{\partial P}{\partial t} = \mathscr{D}\Delta P. \tag{87}$$

This well-known equation holds in many situations, such as the Brownian motion of particles diffusing in a fluid, the diffusion of a fluid in another fluid (in this case P is replaced by the concentration of the diffusing fluid), and also the propagation of heat (in this case P is replaced by the temperature).

Conversely, when the diffusion coefficient vanishes, the Fokker–Planck equation is reduced to the continuity equation,

$$\frac{\partial P}{\partial t} + \text{div}(Pv) = 0. \tag{88}$$

C.2. Continuity equation

Let us now make, in the general case where v and $\mathscr{D}$ are *a priori* non-vanishing, the change of variable

$$V = v - \mathscr{D}\nabla \ln P, \tag{89}$$

that is inspired by the relation between V, v_+ and P in the scale-relativistic equations (but now these variables describe quite classical processes).

We shall first prove that the new velocity field $V(x, y, z, t)$ is solution of the standard continuity equation. Indeed, by using the Fokker–Planck equation and by replacing V by its above expression, we find

$$\frac{\partial P}{\partial t} + \text{div}(PV) = \{\mathscr{D}\Delta P - \text{div}(Pv)\} + \text{div}(Pv) - \mathscr{D}\,\text{div}(P\nabla \ln P). \tag{90}$$

Finally, the various terms cancel each other and we obtain also the continuity equation for the velocity field V:

$$\frac{\partial P}{\partial t} + \mathrm{div}(PV) = 0. \tag{91}$$

Therefore the diffusion term has been absorbed in the re-definition of the velocity field.

C.3. Euler equation

Let us now go one step further in such a fluid-like description of the diffusing motion. The question that we now want to address here is: What is the form of the Euler equation for the velocity field V?

C.3.1. Case of vanishing mean velocity

Let us calculate its total time derivative in the simplified case $v = 0$:

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \left(\frac{\partial}{\partial t} + V.\nabla\right)V = -\mathscr{D}\frac{\partial}{\partial t}\nabla \ln P + \mathscr{D}^2(\nabla \ln P.\nabla)\nabla \ln P. \tag{92}$$

Now, since $\partial\nabla \ln P/\partial t = \nabla\partial \ln P/\partial t = \nabla(P^{-1}\partial P/\partial t)$, we can make use of the diffusion equation so that we obtain:

$$\left(\frac{\partial}{\partial t} + V.\nabla\right)V = -\mathscr{D}^2\left\{\nabla\left(\frac{\Delta P}{P}\right) - (\nabla \ln P.\nabla)\nabla \ln P\right\}. \tag{93}$$

C.3.2. Basic remarkable identity

In order to write this expression in a more compact form, we shall use the following remarkable identity

$$\frac{1}{\alpha}\nabla\left(\frac{\Delta R^\alpha}{R^\alpha}\right) = \Delta(\nabla \ln R) + 2\alpha(\nabla \ln R.\nabla)(\nabla \ln R). \tag{94}$$

This identity plays a fundamental role in all the theory. It is a generalization of the remarkable identity that is at the heart of the scale-relativistic derivation of the Schrödinger, Klein–Gordon and Dirac equations (Nottale, 1993, 1994a; Célérier and Nottale, 2004).

C.3.3. Diffusion potential

Let us now apply this relation in order to give a simplified form to the Euler equation for the velocity field V. By now writing the remarkable identity Eq. (94) for $R = P$ and $\alpha = 1$, we can replace $\nabla(\Delta P/P)$ by $\Delta(\nabla \ln P) + 2(\nabla \ln P.\nabla)\nabla \ln P$, so that Eq. (93) becomes

$$\left(\frac{\partial}{\partial t} + V.\nabla\right)V = -\mathscr{D}^2\{\Delta(\nabla \ln P) + (\nabla \ln P.\nabla)\nabla \ln P\}. \tag{95}$$

The right-hand side of this equation comes again under the identity (94), but now for $\alpha = \frac{1}{2}$. Therefore we finally obtain the following form for the Euler equation of the velocity field $V = v - \mathscr{D}\nabla \ln P$:

$$\left(\frac{\partial}{\partial t} + V.\nabla\right)V = -2\mathscr{D}^2\nabla\left(\frac{\Delta\sqrt{P}}{\sqrt{P}}\right). \tag{96}$$

The comparison of this result with the quantum result (Eq. (43)) in the free case, namely

$$\left(\frac{\partial}{\partial t} + V.\nabla\right)V = +2\mathscr{D}^2\nabla\left(\frac{\Delta\sqrt{P}}{\sqrt{P}}\right), \tag{97}$$

shows that they have exactly the same form, except for the transformation $\mathscr{D}^2 \leftrightarrow -\mathscr{D}^2$.

The case of a non-vanishing mean velocity field can also be treated and leads to similar results.

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