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Review

Scale relativity theory and integrative systems biology: 1 Founding principles and scale laws

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Abstract

In these two companion papers, we provide an overview and a brief history of the multiple roots, current developments and recent advances of integrative systems biology and identify multiscale integration as its grand challenge. Then we introduce the fundamental principles and the successive steps that have been followed in the construction of the scale relativity theory, and discuss how scale laws of increasing complexity can be used to model and understand the behaviour of complex biological systems. In scale relativity theory, the geometry of space is considered to be continuous but non-differentiable, therefore fractal (i.e., explicitly scale-dependent). One writes the equations of motion in such a space as geodesics equations, under the constraint of the principle of relativity of all scales in nature. To this purpose, covariant derivatives are constructed that implement the various effects of the non-differentiable and fractal geometry. In this first review paper, the scale laws that describe the new dependence on resolutions of physical quantities are obtained as solutions of differential equations acting in the scale space. This leads to several possible levels of description for these laws, from the simplest scale invariant laws to generalized laws with variable fractal dimensions. Initial applications of these laws to the study of species evolution, embryogenesis and cell confinement are discussed.

Keywords: Systems biology; Scale relativity; Scale laws; First principles; Multiscale integration

Contents

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1. Introduction

There is nothing more practical than a good theory—Paul Dirac.

The purpose of this series of two companion papers is to discuss the challenges systems biology is facing in its recent developments and how the conceptual framework of scale relativity theory could contribute to address them. It is intended to reflect on the dialog developed between a biologist (CA) and an astrophysicist (LN), and to stimulate further interactions at the interface of physics and biology and beyond. We have attempted therefore to provide a wide readership with what we found necessary and sufficient for a mutual understanding of our arguments and proposals, including basic descriptions of concepts, methods and mathematical developments. Being aware that the fragmentation of sciences in the modern era has led to a large extent biologists to be trained without the deep exposure to the mathematical and formal tools which is typical of physicists, chemists, computer scientists and engineers, we have attempted to describe the essential

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steps in the construction of increasingly complex scale laws in a manner which, for the most part, requires only basic knowledge of the classical differential calculus.

1.1. Motivation

Although the philosophy of nature since the ancient times attempted to describe and understand living creatures as well as the surrounding world, biology is, from an historical point of view, a much younger discipline than chemistry or physics. It was only at the onset of the 19th century that the word biology was introduced by Lamarck and others. During the past two centuries, biology matured through the collection of many observations on the composition and behaviour of the wide range of living systems existing on Earth, and their integration into elements of theories of life. For example, according to the cell theory, all living creatures are made of cells and all cells derive from other cells. The questions of their origin, development and evolution became the subject of very active fields of inquiry. Introduction of the experimental method into biology and medicine led to the development of physiology, microbiology, biochemistry and genetics, each of which relied to some extent on inputs from chemistry or physics to identify its operating principles. Thus biology rapidly became fragmented into a set of sub-disciplines dealing on one hand with particular levels of organization such as ecosystems, organisms, cells and their molecular constituents, and on the other hand with general processes such as their evolution, development and behaviour over different time ranges and in different contexts. As a result, biology has been equipped with numerous axiomatic principles and laws of the cellular, biochemical, evolutionary and developmental theories lacking a common basis.

With the rapid development of a variety of powerful analytical methods and tools over the past century, including high-resolution imaging from entire organisms to molecules, and the high-throughput sequencing and functional genomics methods, it was hoped that the increasing ability to exhaustively describe living systems in terms of their constituents and interrelationships would lead rapidly to a better understanding of their behaviour and to a wide range of useful applications in health, agriculture and environment management. There is, however, a growing recognition that, despite the very significant achievements of the past decades, this accumulated knowledge is necessary but insufficient to reach these goals because of the complex nature of biological systems and the lack of an appropriate conceptual framework enabling integration and interpretation of the diverse data types into a coherent theory of life. To a large extent, both the theoretical elements and the experimental methods have been developed along their own paths, thus offering the prospect of mostly local, *ad hoc* models and solutions to biological questions. The efforts to use a wide range of advanced mathematical and computational methods inherited from computer, information, communication and engineering sciences for the modelling and simulation of the behaviour of biological systems face the same intrinsic limitations, particularly since they are often based on incompatible principles.

It is our contention that integrative systems biology is a research strategy intended to deal with these fundamental issues by developing an iterative scheme of experimental design, data collection, modelling and interpretation addressing specific biological questions, and that to be successful, it needs to be founded on the same first principles as chemistry and physics. One of us has proposed as working hypotheses and conjectures for integrative systems biology that living systems have the ability to organize themselves as the result of a conjunction occurring through an interface between the variable part of a mostly stable physical organization, which can be compared to a scaffolding, and the stable part of a network of small fluctuations (Auffray et al., 2003a). In this context, complex biological systems are viewed as operating in a space with a variable number of multiple biological dimensions designated as "biological space–time" which remains to be formally and operationally defined. The analytical reductionist framework based on the Cartesian precepts of objectivity, decomposition, causality and exhaustivity has enabled mostly identification of the stable physical scaffolding of living systems. It has, however, provided only a limited ability to integrate the multitude of low intensity events occurring at various levels of organization. In order to enable development of a system-level understanding in biology, it should be now complemented by systemic modelling using also conjunctive rather than only disjunctive logic, thus implementing the additional precepts of contextualization, relatedness, conditionality and pertinence.

As a matter of fact, classical analytical methods tend to disjoin the elements of description in order to simplify representation and understanding of a system, whereas systemic modelling methods tend to focus on

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system properties resulting from the conjunction of element characteristics, admitting the possibility of superimposed states, thus providing access to a more global understanding.

Indeed, the small fluctuations of multiple biological parameters, which are inaccessible to currently available tools, may be the major determinants of the behaviour of biological systems because they convey collectively the most important part of biological information. Detection and interpretation of small changes of these low intensity signals will require the development of a new conceptual and practical framework combining in an iterative mode systemic modelling of biological systems, to generate hypotheses, together with a high level of standardization of high-throughput experimental and computational methods and platforms enabling reliable cross comparisons through a shared information system, to test them.

In order to overcome the hurdles that preclude a full implementation of the integrative systems biology approach across multiple levels of organization and time frames, we have explored in depth the potential represented by the theory of scale relativity developed over the past two decades by one of us (Nottale, 1989, 1993, unpublished) to provide the required conceptual and mathematical framework to found a theory of life based on the same first principles as physics and chemistry. It is the purpose of this paper series to provide an overview of the multiple roots from which integrative systems biology is rapidly developing and the challenges faced, and a guide through the different steps of development of the theory of scale relativity and its initial applications to biological systems.

The choice of the scale relativity theory for founding a possible future mathematical biology represents itself a double challenge. Indeed, while there is general acceptance among physicists of the theories of motion relativity, the new theory of scale relativity is still a matter of research and discussion. On the other hand, while Einstein's general relativity is concerned with the foundation of gravitation and therefore has little direct biological applications, the obvious and general occurrence of scales in biological systems supports the proposal that such a fundamental theory of scales in nature, despite its novelty, could be particularly well adapted to biology. As we are going to see, the scale relativity theory has an intrinsic capacity to decompartmentalize the scientific disciplines, so that its applications in biology may, in the end, be also helpful in deciding on its value.

1.2. A brief history of integrative systems biology

The current development of systems biology since it was given its name four decades ago (Mesarovic, 1968; Wolkenhauer and Mesarovic, 2005) represents a new phase of a recurrent theme in the development of science during the past four centuries, sustained by repeated and fruitful encounters of natural sciences with physical, mathematical and engineering sciences in an accelerating pace (reviewed in Kitano, 2002; Csete and Doyle, 2002; Hastings and Palmer, 2003; Brent, 2004; Cohen, 2004; Coveney and Fowler, 2005; Doyle and Stelling, 2006).

1.2.1. Physiology, genetics and molecular biology

William Harvey introduced a numerical calculation to establish the parameters for a model of blood circulation (Harvey, 1628), predicting the existence of capillaries at the junction of arteries and veinules, which were observed much later by Spallanzani and others thanks to the invention of the microscope, thus founding the science of physiology.

Gregor Mendel based his work on heredity and development on the mathematical and physical thinking he had acquired from Doppler and others to write the first general equation of biology, providing the foundation of genetics (Orel, 1996; Auffray, 2005). His formal model of inheritance was refined through the development of statistics by Pearson and Fischer, and established on cellular and biochemical grounds by Morgan, Avery and many others (reviewed in Sturtevant, 1965).

The development of molecular biology was triggered by the physicists Dellbrück and Schrödinger who provided the framework for a definition of the gene at the atomic level (Schrödinger, 1944), which was soon after established as a double helix of anti-parallel complementary strands based on high-precision X-ray diffraction diagrams interpreted through Fourier transformation presented in a series of papers in *Nature* (Watson and Crick, 1953a; Wilkins et al., 1953; Franklin and Gosling, 1953a,b). The structure of DNA immediately suggested possible explanations for a number of important biological phenomena (Watson and

Crick, 1953b). This led, through a wide range of biochemical experiments in a variety of microbial, plant and animal species, to the discovery of the genetic code and the universal mechanisms of replication of DNA, transcription and translation for the formation of proteins and the expression of biochemical and cellular activities.

Each and all these examples follow a common scheme combining the formulation of a precise question for interrogation of a biological system based on advances in the ability to measure biological parameters with incorporation of the results into a predictive model, thus complying with both the analytical and the systemic precepts (Fig. 1). They can be considered as prominent forerunners of systems biology which has thus multiple converging roots in physiology, genetics and molecular biology, each made possible by the conjunction of novel technologies and mathematical models.

Thus a long phase of inquiry-based research which spanned almost 350 years after the invention of the microscope was followed by half a century dominated by technology driven advances as discussed below.

1.2.2. Genetic engineering, functional genomics and computational biology

Models were proposed soon after the discovery of the double helix for the regulation of biological phenomena such as the diauxic shift in bacteria through control and coordination of the expression of genes physically and functionally linked into an operon, based primarily on feedback circuits identified through precise experimental measurements of cell growth in a chemostat (Novick and Weiner, 1957). This and similar observations (e.g., Yates and Pardee, 1957) in turn triggered elucidation of genetic regulatory mechanisms, and the prediction and discovery of the messenger RNA as an intermediate between the gene and the encoded protein (Jacob and Monod, 1960).

Systems biology in its present form (reviewed in Brent, 2000; Auffray et al., 2003b; Hood et al., 2004; Kirschner, 2005) is also the result of a number of initial attempts since the 1940s to use control and system theory in biology since the pioneering works on general system theory (von Bertalanffy, 1945; Rosen, 1958; Simon, 1981); on cybernetics (Wiener, 1948; Ashby, 1956); on modelling and control of morphogenesis and metabolic networks (Turing, 1952; Waddington, 1957; Mitchell, 1961; Britten and Davidson, 1969; Guyton et al., 1972; Kacser and Burns, 1973; Savageau, 1976) and to take into account the non-linear dynamics behaviour of complex biological systems (Glansdorff and Prigogine, 1971; Kauffman, 1993; Kitano, 2001; Wolkenhauer, 2001; Kholodenko and Westerhoff, 2004; Westerhoff and Palsson, 2004).

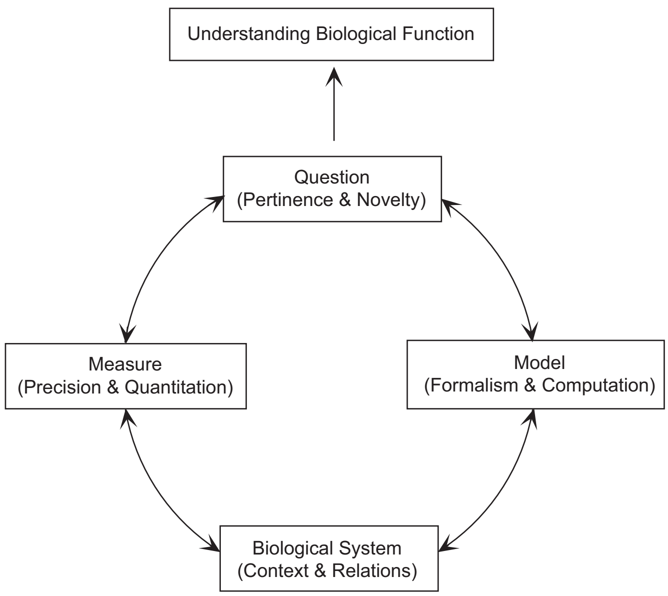

Fig. 1. The basic middle-out scheme of inquiry on biological systems, combining in an iterative loop analytical tools and systemic principles. The arrows indicate the two complementary paths contributing to biological understanding: clockwise for the top-down approach, counter-clockwise for the bottom-up approach.

These efforts were hampered by the lack of appropriate technologies to collect the precise measurements required to feed, test and validate the large number of possible models generated. They were also overshadowed by the spectacular advances of mainstream molecular biology. These obstacles could be overcome in part with the development of recombinant DNA technology, which formed the basis for genetic engineering, and of high throughput sequencing and functional genomics for the assessment of extensive sets of transcripts (transcriptomes), proteins (proteomes), metabolites (metabolomes) and functions (physiomes).

These tools take advantage of advances in a variety of engineering disciplines including optics, electronics, robotics and fluidics, combined with information, communication and formal computer and mathematical sciences. They made it possible to study the structure of the genome of man and many microbial, plant and animal species on a global scale, and to start to address the behaviour of biological systems in natural and perturbed conditions in a renewed framework. This explains the recent explosive development of systems biology as a discipline on its own and the extension of genetic engineering into synthetic biology (reviewed in Arkin, 2001; Benner and Sismour, 2005; Church, 2005; Endy, 2005; Andrianantoandro et al., 2006; Drubin et al., 2007; Tyo et al., 2007), based on growing interdisciplinary interactions and individuals, as demonstrated by the creation of research centres and institutes, journals and conferences worldwide (Aderem, 2005; Eddy, 2005; Liu, 2005; Tadmor and Tidor, 2005; Ideker et al., 2006).

1.2.3. The grand challenge of systems biology: multi-scale integration

In practice, the integrative systems biology approach can be summarized through the following steps: formulate a specific biological question; define the components of a biological system and collect the biochemical and genetic data which are relevant to address the question; use them to formulate an initial model of the system and make predictions on its behaviour; systematically perturb the components of the system and study the result; compare the observed responses to those predicted by the model; refine the model so that its predictions fit best to the experimental observations; conceive and test new experimental perturbations to distinguish between multiple competing hypotheses (Ideker et al., 2001; Auffray et al., 2003b; Klipp et al., 2005).

The question as to where and how to initiate the process of an integrative systems biology inquiry has triggered intense scientific, technological and epistemic debates (Cornish-Bowden et al., 2004; van Regenmortel, 2004; O'Malley and Dupré, 2005; Ahn et al., 2006a,b; Wolkenhauer et al., 2007) between the proponents of a bottom-up, data-driven approach aimed at inferring model structures from high-throughput experimental data and those supporting a top-down approach based on conceptual models derived from engineering and computational science design principles (Alon, 2003; Bray, 2003; Ideker and Lauffenburger, 2003; Tyson et al., 2003; Barabasi and Oltvai, 2004; Werner, 2005; Zhuravel and Kaern, 2005; Wilson, 2007). As discussed above and shown in Fig. 1, these approaches are not contradictory but must be combined effectively together in a manner which depends primarily on the question addressed. This integrative strategy corresponds to the "middle-out" approach to biological systems proposed by Brenner (1998) and Noble (2002a, 2006). It expresses in a concrete manner the principle that there is no priviledged level in a biological system which would determine events occurring at the other levels, only interactions between multiple levels (Noble, 2006). As a result, the levels of input and input into and from the system investigated are primarily defined by the question addressed.

It is beyond the scope of these papers to examine in detail the recent progress in understanding biological phenomena on a systemic level. It is not in our intention to dismiss the value of reported advances such as extracting functional and regulatory order through integration of multiple functional genomics data, literature mining and systemic modelling (e.g., Weinstein et al., 1997; Davidson et al., 2002; Shannon et al., 2003; Carpenter and Sabatini, 2004; Fernie et al., 2004; Imanishi et al., 2004; Nicholson et al., 2004; Hwang et al., 2005; Imbeaud and Auffray, 2005a; Graudens et al., 2006; Jensen et al., 2006; Vidal, 2005; Reed et al., 2006). Progress has been made in the development of a theory of robustness and fragility (reviewed in Carlson and Doyle, 2002; Selinger et al., 2003; Kitano, 2004; Liu and Lemberger, 2007), and assessment of the role of modularity and coupling of networks and pathways in biological systems (e.g., Kitami and Nadeau, 2002; Milo et al., 2002; Isaacs et al., 2003; Papin et al., 2004; de Silva and Stumpf, 2005; Locke et al., 2005; Mazurie et al., 2005; Prill et al., 2005; Sprinzak and Elowitz, 2005; Kafri et al., 2006; Kitayama et al., 2006; Maciag et al., 2006; Radulescu et al., 2006; Hayete et al., 2007; Zhu et al., 2007).

A variety of experimental schemes have highlighted the role of stochastic fluctuations or ‘‘biological noise’’ (McAdams and Arkin, 1997; Hasty et al., 2000) in gene expression networks at the molecular (Ozbudak et al., 2002; Blake et al., 2003; Fraser et al., 2004), modular (Rives and Galitski, 2003; Orell et al., 2005; Leake et al., 2006) or cellular (Brandman et al., 2005; Rosenfeld et al., 2005) levels in driving the behaviour of biological systems (reviewed in Raser and O'Shea, 2005). Realistic integrated models of functioning heart, lung and muscle have been developed in the frame of the Physiome Project (Bassingthwaighte, 2000; Noble, 2002a,b; Hunter and Borg, 2003; Hunter and Nielsen, 2005). Systems biology approaches are now endorsed by the biotechnology and pharmaceutical industry and start to impact on the drug discovery process (Hood and Perlmutter, 2004; Butcher et al., 2004; Bugrim et al., 2004; Searls, 2005; Cho et al., 2006; Loging et al., 2007).

Rather, we would like to point to the fact that in each of its implementations, the systems biology research strategy relies on *ad hoc* combinations of measurement technologies and formalisms for mathematical modelling. It is now widely recognized that in most instances, current technologies are either inadequate or used with improper experimental design (Ransohoff, 2004, 2005; Ioannidis, 2005) to provide the type of data and level of precision required to build and assess the validity of models accounting for the dynamic behaviour of biological systems.

Most importantly in our view, in many instances of successful advances in systems biology programmes, although the multiple hierarchical organization levels (from molecules to ecosystems) and time frames (from milliseconds to billions of years) are recognized as essential features of living systems, the use of multiple, fundamentally incompatible mathematical and computer formalisms for modelling, e.g., morphogenesis (Chaturvedi et al., 2005), or cellular pathways (Takahashi et al., 2004; Hua et al., 2006) makes data integration and model comparison or coupling difficult if not impossible. The increasing complexity of the models, some of which are based on thousands or even tens of thousands of equations, make numerical simulation computationally intractable, and it is also severely limited by available computing power despite its exponential growth within computer grids (Coveney and Fowler, 2005; Hunter and Nielsen, 2005). Further technological developments of nanomolecular technologies (Hong et al., 2002), high-resolution imaging (Phelps, 2004), markup languages for biochemical modelling such as CellML (Lloyd et al., 2004) or SBML (Hucka et al., 2003), and mathematical and computational modelling formalisms (Thieffry et al., 1998; Smolen et al., 2000; Hasty et al., 2001; Regev et al., 2001; Errampalli et al., 2004; Friedman, 2004; Roux-Rouquié et al., 2004; Kitano et al., 2005; BioUML, 2006; Calzone et al., 2006) are in order and the focus of intense community efforts for data integration, standardization and interoperability under quality assurance (Ashburner et al., 2000; Cassman, 2005; Imbeaud and Auffray, 2005b; Brazma et al., 2006; Swertz and Jansen, 2007).

For all these reasons, in the absence of an appropriate coherent theoretical framework enabling one to take into account effects occurring on the multiple scales which are typical of biological systems (Fig. 2), systems biology approaches could yield yet another layer of description with limited explanatory power. Indeed, allometric scale laws that relate mass and energy metabolism in biological systems according to quarter-power laws have been uncovered since the pioneering work of Kleiber in the 1930s, and recently shown to extend across 27 orders of magnitude of size from molecules to entire organisms, suggesting elements for a unifying theory of biological structure and organization (reviewed in Brown and West, 2000; West et al., 2002; West and Brown, 2005), but without relating them to fundamental principles. We introduce in what follows scale relativity theory as a possible solution to overcome these essential difficulties and to found the development of integrative systems biology on first principles.

1.3. The theory of scale relativity

The theory of scale relativity is an extension of the theories of relativity, achieved by applying the principle of relativity not only to motion transformations, but also to scale transformations of the reference system. Recall that, in the formulation of Einstein (1916), the principle of relativity consists of requiring that ‘‘the laws of nature be valid in every systems of coordinates, whatever their state’’. Since Galileo, this principle had been applied to the states of position (origin and orientation of axes) and of motion of the system of coordinates (velocity, acceleration). These states are characterized by their relativity, namely, they are never definable in a

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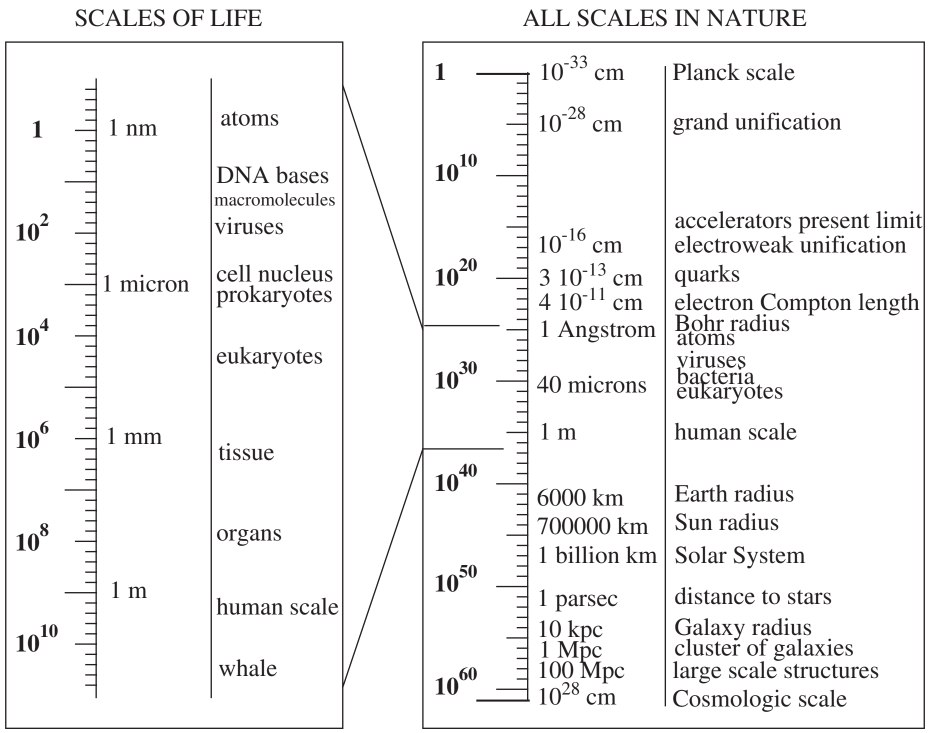

Fig. 2. Scales in nature. The range of biological scales (left), embedded in the range of physical scales (right).

absolute way. This means that the state of any system (including reference systems) can be defined only relatively to another system.

One of us (LN) has suggested that the observation scale, i.e., the space and time resolution at which a system is observed or experimented, should also be considered as characterizing the state of reference systems. It is an experimental fact known since Greek philosophers that the scale of a system can be defined only in a relative way: only scale ratios do have a physical meaning, never absolute scales. This led to the proposal that the principle of relativity should be generalized to apply also to the transformations of the scale of reference systems (Nottale, 1989, 1992, 1993). In this new approach, one re-interprets the length and time resolutions, not only as a property of the measuring device and/or of the measured system, but more generally as a property that is intrinsic to the geometry of space–time itself: in other words, space–time is fractal. The principle of relativity of scale then consists of requiring that ‘‘the fundamental laws of nature apply whatever the state of scale of the coordinate system’’.

Under such a view, one comes back to the primacy of space–time variables, which recover their status of fundamental variables, at the level of positions and instants (position, orientation and motion relativity), but also of space and time intervals (scale relativity). The other physical quantities, energy, momentum, angular momentum, etc. are derived quantities which are also relative as a consequence. Note also that the new scale variables (called ‘‘resolution’’ under a new general acceptance) are not ‘hidden variables’, but clearly known and observed variables, including in quantum experiments, the meaning of which is only reinterpreted in the new theory. As we shall see in detail in the companion paper, the non-differentiability of a continuous space–time implies a fundamental undeterminism of its geodesics, from which one may derive the quantum theory and its various properties (Nottale, 1993, unpublished; Célérier and Nottale, 2004), including the classical to quantum transition at the de Broglie scale, identified to a spontaneous non-fractal to fractal transition (see Section 3.2.4).

This first principle allows one to generalize the current description of the geometry of space–time, which is usually reduced to at least two-time differentiable manifolds. Such a generalization of physics consists of

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abandoning the hypothesis of differentiability of space–time coordinates. This means to consider general continuous geometries including as a subset the usual differentiable ones, and therefore all the Riemannian geometries that underlie Einstein's generalized relativity of motion. In such an approach, the standard classical physics will be naturally recovered as special cases.

New physics thus emerges in this extended framework. Indeed one can prove the fundamental founding theorem (Nottale, 1993, 1996; Ben Adda and Cresson, 2000; Cresson, 2003, Nottale, unpublished) according to which a continuous and non-differentiable curve is fractal, under Mandelbrot's general definition of this concept (Mandelbrot, 1975, 1982): namely, *the length of a non-differentiable curve acquires an explicit dependence on resolutions and diverges when the resolution interval tends to zero* (Nottale, 1993; Ben Adda and Cresson, 2000). This fundamental result can be extended to space, and more generally to space–time, and it therefore supports the early proposals to consider fractal space–time (Ord, 1983; Nottale and Schneider, 1984; Nottale, 1989) as a new geometric tool for a generalized physical description, in particular of quantum mechanical laws.

Taking into account the scale dependence allows one to solve the non-differentiability problem, and to describe non-differentiable functions in terms of differential equations (Nottale, 1989). Indeed, in such a framework, the differential element $\mathrm{d}x$, which describes the way in which a variable is divided from the theoretical viewpoint of the differential calculus, can now be treated as a variable on its own, rather than only as a vanishing quantity ($\mathrm{d}x \to 0$).

The same is true in the experimental case of the resolution interval ε that is characteristic of any measurement apparatus. Both quantities, the small interval $\mathrm{d}x$ used in the theoretical description, and the experimental resolution ε which intervenes in the final comparison between theory and experiment, are considered in the new framework as characterizing the state of scale of the system under consideration.

A non-differentiable function $f(x)$, which is, by definition, such that its derivative $f'(x)$ does not exist, can now be theoretically described by a fractal function $f(x, \mathrm{d}x)$ which is an explicit function of the scale interval $\mathrm{d}x$. (In the experimental case, one considers quantities that are explicitly dependent on the resolution interval ε, i.e., $f(x, \varepsilon)$). It is only at the limit $\mathrm{d}x \to 0$ that the derivative $\mathrm{d}f/\mathrm{d}x$ does not exist. Instead of considering only this limit (that is actually unreachable), as in the usual calculus, the new scale relativity method amounts to describe in detail what happens when $\mathrm{d}x$ is continuously decreasing.

Namely, the ordinary function $f(x)$ is identified with $f(x, 0)$. But while $\partial f(x, 0)/\partial x$ does not exist (here ∂ represents a partial derivative), the generalized fractal function $f(x, \mathrm{d}x)$ is doubly differentiable when $\mathrm{d}x \neq 0$, with respect to the standard coordinate x, but also with respect to the new scale coordinate $\mathrm{d}x$. Therefore, a full physical description of this function can then be obtained by a double differential calculus, in the standard position space (x) and in the new internal space of the scale variables ($\mathrm{d}x$), that we have called "scale space". From the experimental viewpoint, this scale space is the space of resolution intervals $\{\varepsilon\}$.

In this context the scale space is considered to be physical, with the consequence, as we shall see, that the full description of a fractal space–time and of its geodesics (which are the minimized paths) includes a twin-coupled description of the geometries of the scale space and of space–time (of positions and instants).

1.4. Definitions

Let us first give some definitions of physical concepts whose generalization may be relevant for inclusion of biology in a scale-relativistic framework based on the same first principles as physics.

1.4.1. Space

The concept of space (more generally, of space–time) is defined by the geometry of the interrelations between objects. The space is the container while the objects are contained. But the space itself is neither an object, nor a substance. It is the relation between the objects, namely, it is defined from objects, but is intrinsically independent of them. For example, a two-dimensional Euclidean space (of positions) is defined by the Pythagoras relation between three points A, B and C forming a right-angled triangle, $AC^2 = AB^2 + BC^2$. In order to verify such a relation, one needs to materialize the three points by three (point-like) "objects", but the relation is independent of the objects chosen. In modern differential notation, it leads to define the length $\mathrm{d}l$ in terms of its projected Cartesian coordinates $\mathrm{d}x$ and $\mathrm{d}y$ as $\mathrm{d}l^2 = \mathrm{d}x^2 + \mathrm{d}y^2$, which is invariant under a rotation while $\mathrm{d}x$ and $\mathrm{d}y$ vary.

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1.4.2. Metric potentials

This relation can be generalized to the three dimensional length, $dl^2 = dx^2 + dy^2 + dz^2$, then to the invariant proper time of special relativity, $ds^2 = c^2dt^2 - (dx^2 + dy^2 + dz^2)$, where c is the velocity of light. This relation defines the metric element of a four-dimensional Minkowskian space. Einstein's theory of general relativity is based on an additional generalization of this relation: in this theory, the geometry of space–time is no longer flat (i.e., Euclidean or pseudo-Euclidean as in the Minkowskian metric of special relativity) but becomes curved. The invariant proper time now involves non-constant coefficients in the definition of the metric, namely,

$$ds^2 = \sum_{ij} g_{ij}\, dx^i dx^j \tag{1}$$

(where the indices i, j run from 0 to 3). The coefficients g_{ij} are now functions of the coordinates, i.e. $g_{ij} = g_{ij}(x, y, z, t)$ and are called metric potentials. This name comes from the fact that they generalize the Newtonian potential of gravitation.

1.4.3. Geodesics

The concept of geodesics plays a fundamental role in space–time theories of relativity. It designates (by extension) the shortest paths in any geometry. For example, the geodesics on the Earth surface are the great circles. Geodesics play a central role in geometric theories of space–time, since the trajectories of free particles in a given geometry are given by them. They thus generalize the straight line, which is the trajectory of free inertial Galilean motion, and may indeed be identified with the geodesics of an Euclidean space.

1.4.4. Dynamics

In physics, dynamics is the study of systems which are subjected to forces. Since Galileo, it is known that motion does not need any force to be continued (this is the law of inertia), so that the effect of a force is to change motion, according to Newton's fundamental law of dynamics, $F = m\,dv/dt$. In the theory of scale relativity, we are led to identify self-similar systems characterized by a constant fractal dimension to the equivalent for scale laws of what is inertia for motion laws, and, therefore, by extension, to consider deviations from self-similarity as coming under a ''scale dynamics'' (i.e., distorsion forces in scale space).

1.4.5. Dimension

The concept of dimension has been generalized by Mandelbrot (1975, 1982) to fractal dimensions. Fractal dimensions do not take the place of topological dimensions, but are complementary to them. Topological dimensions are only integer numbers: a point or a dust of points has topological dimension $D_T = 0$, a curve $D_T = 1$, a surface $D_T = 2$, a volume $D_T = 3$, etc. Two objects or systems have the same topological dimensions if one can define a continuous and one-to-one transformation between them.

The fractal dimensions that we shall consider here are essentially ''covering dimensions'' (i.e., the properties of a system covered with balls of varying sizes ε depend on the size of the balls). They mainly correspond to the standard ways of dissecting a measured variable in physics in terms of small elements, namely: (i) in the experimental and observational domain, the measured value x is related to the resolution δx of the measurement device; (ii) in the theoretical domain, one considers small differential increments of variables, $x \to x + dx$. The main method of scale relativity consists of treating these scale variables as explicit variables, instead of making either $\delta x \to 0$ or $dx \to 0$ and of letting them disappear from the equations (which is possible only in the differentiable case). A fractal coordinate X is therefore an explicit function of δx or dx, i.e., $X = X(\delta x)$ [experimental description] and $X = X(dx)$ [theoretical description].

2. Founding first principles and methods

2.1. Evolution of concepts in physics

Let us first briefly recall some features of the evolution of ideas in physics, by taking the theories of gravitation as an example. This example is chosen to illustrate the kind of evolution of life sciences that one may expect in the future. Indeed, the physical approach has shifted from mainly descriptive modelling to genuine predictive theories founded on first principles, and from a way of investigation based on setting *a priori* hypotheses and/or postulates (such as Newton's theory of gravitation and today's quantum mechanics) to a new way of thinking pioneered by Einstein, that consists of actively abandoning fundamental axioms underlying previous physical theories. For example, construction of Einstein's theory of general relativity was made possible by the assumption that the general belief that the geometry of space could be nothing else than Euclidean could be abandoned. In this theory, flat space becomes a very particular case of all the possible curved geometries of space–time.

Such an approach proceeds through important generalizations which involve the creation of new concepts aimed at describing the multiple structures which are expected to emerge. It is an extension of the frame of work and of thought, in which the hypotheses abandoned, which seemed to be unavoidable in the previous paradigm, are revealed for what they actually were, namely, unnecessary blocking constraints (for example, before the discovery of curved geometry by Gauss, Bolyai and Lobachevsky, one could not even imagine that geometry could be anything else than Euclidean). This extended framework leads to a deeper understanding of nature, since the structures which are explained by this method are understood as the manifestation of principles of the greatest generality: hence, in general relativity, the various aspects of gravitation are understood as the multiple manifestations of the curvature of space–time; in the theory of scale relativity, it is the implicit axiom of differentiability, that underlies the classical theoretical description of all natural sciences, that is abandoned.

2.1.1. Ptolemeus' model

The first step in our knowledge of the motion of astronomical bodies was merely descriptive. Ptolemeus' epicyclic model of planetary motions was intended to account for the direct positions of planets in the sky. As there are as many parameters as degrees of freedom in such a model, no genuine understanding can be gained, and it only describes in a direct way the observational data.

2.1.2. Kepler's model

The second step was Kepler's model of planetary motion that relies on Copernicus' discovery that the planets follow orbits around the Sun. Its ability to synthesize a huge amount of observational data is already extraordinary. Indeed, it allows a profound understanding of many features of planetary motion, but it nevertheless remains a model since the three Kepler laws (elliptical orbits, law of area, relation between periods P and semimajor axes) are inferred from an analysis of observations but have no theoretical basis, although Kepler himself did look for a central force.

2.1.3. Newton's theory

The third step was Newton's universal theory of gravitation. The conceptual framework becomes in this case profound enough that it deserves the name of theory, although, as remarked by Newton himself, the basis of the theory remains axiomatic and badly founded. In this framework the three Kepler laws are now proved, understood and generalized to parabolic and hyperbolic motion thanks to the introduction of the gravitational force, and the theory acquires a large predictive power (as exemplified, e.g., by the prediction of the planet Neptune). But the expression for the force, proportional to mass and inversely proportional to the square of the distance, remains a postulate of unknown origin.

2.1.4. Field theory

The fourth step was field theory (Poisson, Laplace, Maxwell, Lorentz). The concept of force, which was defined locally between two bodies, is extended to that of field, which is now assumed to be created by a body (namely, by its active "charge"), fills the whole space, and is felt by another body provided it has a passive charge. The description of this force field now involves two concepts, the field itself and the potential, from which the field derives. The theory now includes field equations (Poisson equation, generalized to Maxwell equations for electromagnetism) and motion equations, which amount to the fundamental equation of dynamics under the force F (function of the field E and of the velocity u, see Appendix A).

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2.1.5. Relativistic theories

The fifth step was Einstein's general relativity of motion, which is also a relativistic theory of gravitation. In order to better understand its meaning and its profound consequences, let us briefly summarize the evolution of ideas concerning the theories of relativity.

2.1.5.1. Galilean relativity. The first level of motion relativity theories is Galileo's relativity of inertial motion. Galileo has set as a principle that "for all things that participate to it, motion remains perfectly imperceptible and as if it were not". This means that motion is not a property of individual bodies, but a relative property between two bodies. It is not definable in the absence of a reference system, and it therefore disappears in the proper reference system (which participates in it).

2.1.5.2. Special relativity. The second level is Poincaré–Einstein's special theory of relativity of motion. The problem is the same as in Galileo's relativity, namely, to find the laws of transformation of inertial coordinates systems under changes of their relative motion. However, the general solution found by Poincaré (1905) and Einstein (1905a) to this problem, the transformation called "Lorentz transformation" by Poincaré, generalizes the Galileo transformation and includes it as a particular degenerate case (namely, when the velocity of light $c \to \infty$). The physical meaning of this transformation is that space and time are no longer separated as in Galilean relativity, but become subspaces of a four-dimensional space–time (see above the definition of metric). As a consequence motion is nothing but rotations in space–time, which explains the relativistic contraction of length and the dilation of time as projection effects under such rotations and relates the relativity of motion to that of orientation.

2.1.5.3. Einstein's general relativity. The third level is Einstein's general relativity of motion. In this theory, the principle of relativity is applied not only to inertial motion, but also to accelerated motion. This has been made possible by Einstein's discovery of the principle of equivalence, according to which a gravitational field is locally equivalent to an acceleration field. Accelerated motion, which seemed to be definable in an absolute way under Newton's view (through the appearance of inertial forces), revealed itself to be once again only relative to the choice of the reference system. Namely, a body in free fall accelerates toward the Earth in the Earth reference frame, and this acceleration is described as resulting from a gravitational force in Newton's approach, then from the effect of the gravitational potential of the Earth in a field theory approach. But Einstein realized that another body in free fall could as well be taken as the reference system, and that two close bodies in free fall are either at rest, one relative to the other, or in inertial relative motion. Therefore neither the acceleration nor the gravitational force or field constructed from it can be considered any longer to exist in an absolute way. As a consequence Einstein's theory of general relativity of motion is at the same time a theory of the relativity of gravitation, which itself depends on the reference system.

In this theory, the concepts of gravitational force, potential, and field, are replaced by the geometry of space–time. While the Minkowskian space–time of special relativity was still absolute (even though separated space and time became relative) and non-structured (flat, i.e., non-curved), the space–time of general relativity becomes curved and relative to its energy-momentum content. What is called gravitation is nothing but the various manifestation of the curved geometry, while free particles follow the geodesics of the Riemannian space–time constrained by Einstein's field equations.

2.1.6. Quantum theory

A sixth step of the evolution of ideas in physics is the quantum theory. Unfortunately, we can no longer use the example of gravitation to illustrate it, since there is not yet any fully self-consistent quantum theory of gravitation. But quantum theories of the other fundamental fields (strong, weak and electromagnetic) do exist and have yielded extraordinary results. The quantum approach is based on a representation which is completely different from the classical one. The description tool is a wave function (or probability amplitude) whose square of the modulus gives the density of probability of observables. Through this description, elementary particles have a triple aspect of particle, wave and field. This becomes the case also for what was classically considered as the source (fermions, of half-integer spin) or the field (bosons, of integer spin). Therefore, in its framework matter (such as quarks and electrons) has wave properties, as discovered by de

Broglie (1924), while force fields (such as electromagnetism) have particle properties, described by Einstein (1905b) in terms of light quanta (later called photons). The quantum theory has achieved an impressive unification of previously separated concepts. The trouble is that its foundation remains axiomatic: indeed, the mathematical tools and the equations themselves are postulates of the theory instead of being derived from first principles.

2.1.7. *Toward a unified view*

From the viewpoint of this history (strongly summarized above), the theory of scale relativity, as we shall see in what follows, aims at preparing the emergence of a seventh step by unifying the quantum and relativity concepts on the basis of first principles (Nottale, 1993; Célérier and Nottale, 2004; Nottale et al., 2006), in a framework where the quantum fields become themselves manifestations of the geometry of space–time.

2.2. *Founding first principles in relativity theories*

2.2.1. *Prerelativistic optimisation principle: least action and conservation laws*

The whole of theoretical physics (classical and quantum) relies on a fundamental optimisation principle (Landau and Lifchitz, 1966), from which the basic equations of physics can be constructed, under the form of Euler–Lagrange equations, relating fundamental quantities such as energy, momentum and angular momentum. This principle of least action becomes, as we shall recall hereafter, a geodesics principle in the framework of relativity theories (i.e., action is identified with the proper time).

The principle of least action states that there exists some function $L(x, v, t)$ of physical systems, called Lagrange function, the integral of which is the action, $S = \int L(x, v, t)\,dt$, and that the motion of the system is such that it optimizes the value of this action to a constant, minimum value. This is expressed by the fact that the first variation of the action vanishes, i.e., $\delta S = 0$. From this principle, a general form of the equations of motion can be derived as Euler–Lagrange equations, that read (d denoting a total derivative and ∂ a partial derivative)

$$\frac{d}{dt}\left(\frac{\partial L}{\partial v}\right) - \frac{\partial L}{\partial x} = 0. \tag{2}$$

Linked to this principle, one proves the existence of fundamental physical quantities which are subjected to conservation laws. These quantities and their conserved (invariant) character result from symmetries of the underlying basic variables of description (this is Noether's theorem). For example, energy is the conservative quantity that results from the uniformity of time, momentum from the uniformity of space and angular momentum from the isotropy of space. In quantum field theories, the charges are the conserved quantities that result from the symmetries of the phases of the wave function.

2.2.2. *The principle of relativity: covariance, equivalence and geodesics*

Let us now briefly recall the fundamental principles that underlie, since the work of Poincaré (1905) and Einstein (1905a, 1916), the foundation of theories of relativity. We shall express them here under a general form that transcends particular theories of relativity, namely, they can be applied to any state of the reference system (origin, orientation, motion, scale, etc.).

The basic principle is the principle of relativity, which requires that the laws of physics should be of such a nature that they apply for any state of the reference system. In other words, it means that physical quantities are not defined in an absolute way, but are instead *relative to the state* of the reference system (which can be static or dynamic). It is subsequently implemented in physics by three interconnected principles and equipped with their related mathematical tools.

2.2.2.1. The principle of covariance. It requires that the equations of physics keep their form under changes of the state of reference systems. As remarked by Weinberg (1972), it should not be interpreted in terms of simply providing the most general (arbitrarily complicated) form to the equations, which would be meaningless. It rather means that, knowing that the fundamental equations of physics have a simple form in

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some particular coordinate systems, they will keep this simple form when considering more general coordinate systems. With this meaning in mind, two levels of covariance can be defined:

Strong covariance, according to which one recovers the simplest possible form of the equations, which is the Galilean form they have in the vacuum devoid of any force. Under this principle, the equations of motion in general relativity take the free inertial form $\mathrm{D}u_\mu/\mathrm{d}s = 0$, in terms of Einstein's covariant derivative, so they come under strong covariance. Here D denotes the covariant derivative (see its construction in the next section), which is the main mathematical tool of the theory. The vector $u_\mu = \mathrm{d}x_\mu/\mathrm{d}s$ is the four-dimensional velocity, defined in the four-dimensional space–time of general relativity by the index μ running on time and space coordinates, i.e., $(0, 1, 2, 3)$ for (t, x, y, z). The variable s represents the proper time (i.e., the time measured by a clock that is taken along with the system considered) and $\mathrm{d}s$ is the proper time differential element, which is the fundamental invariant of the theory. One of the main tools for implementing strong covariance is the tensor calculus, which is a natural generalization of vectors to several indices, and which allows to compactify the writing of the equations (Alunni, 2001), namely, tensors have a particularly simple way to transform under changes of coordinate systems.

Weak covariance, according to which the equations keep the same, simple form under any coordinate transformation, but not as simple as the free Galilean-like equation. A large part of the general relativity theory is only weakly covariant in this context. For example, Einstein's field equations have a source term, the stress-energy tensor of matter, while Einstein's initial hope was to construct a purely geometric theory in which the sources themselves would be of geometric origin. Another case of weak covariance in general relativity concerns the gravitational fields, which are not tensors, but transform themselves under changes of coordinates in a more complicated, non-linear way.

2.2.2.2. The principle of equivalence. It is a more specific statement of the principle of relativity, when it is applied to a given physical domain. In general relativity, it states that a gravitational field is locally equivalent to a field of acceleration, i.e., it expresses that the very existence of gravitation is relative to the choice of the reference systems, and it specifies the nature of the coordinate systems in which gravitation locally disappears. Therefore, in such an accelerating coordinate system, the field of gravitation has vanished, so that the motion equation naturally takes the strongly covariant form of free motion devoid of any force, i.e., once again $\mathrm{D}u_\mu/\mathrm{d}s = 0$.

In scale relativity, one may make a similar proposal and set generalized equivalence principles according to which the quantum behaviour is locally equivalent to a fractal and non-differentiable motion, while the gauge fields are locally equivalent to expansions or contractions of the internal resolutions.

2.2.2.3. The principle of geodesics. It states that the free trajectories are the geodesics of space–time. It plays a very important role in a geometric relativity theory, since it means that the fundamental equation of dynamics is completely determined by the geometry of space–time, and therefore has not to be set as an independent equation. Moreover, in such a theory the action $\mathrm{d}S$ identifies itself (modulo a constant) with the fundamental metric invariant which is but the proper time $\mathrm{d}s$, namely, $\mathrm{d}S = -mc\,\mathrm{d}s$, so that the action principle becomes nothing else than the geodesics principle. As a consequence, its meaning becomes very clear and simple: the physical trajectories are those which minimize the proper time itself. Moreover, contrarily to what happens in other fundamental physical theories, which need to specify both the field equations and the trajectory (i.e., dynamics) equations (such as in Newton's and Maxwell's theories), once the geometry is known (i.e., the field) the trajectories are also known since the geodesics are completely defined by the geometry.

2.3. Method of scale relativity theory

The basic tool of scale relativity amounts to introduce explicitly scale variables in the physical description. Therefore the method of the scale relativity theory can be decomposed into three steps.

(1) Find the laws of scale at a given point and instant. These laws are obtained as solutions of differential equations acting in scale space (i.e., that describe the effect on physical quantities of an infinitesimal zoom), constrained by the principle of scale relativity.

(2) For each of these underlying scale laws, find the laws of motion in standard space (fundamental equation of dynamics). They are written in terms of a geodesics equation using a covariant derivative tool that includes the effects of non-differentiability and fractality in the differentiation process itself. As we shall see, the laws of motion constructed in this way acquire a quantum-type form.

The concept of covariant derivative is one of the main mathematical tools by which these principles are implemented. This tool includes in an internal way the effects of geometry through a new definition of the derivative, contrarily to the standard field approach whose effects are considered to be externally applied on the system. In general relativity, it amounts to subtracting the geometric effects to the total increase of a vector, leaving only the inertial part, which defines the covariant derivative. One of the most remarkable results of general relativity is that the three principles (strong covariance, equivalence and geodesics) lead to the same form for the motion equation, which is the simple Galileo form of inertial motion of a free body submitted to no force.

The theory of scale relativity follows a similar line of thought, and constructs new covariant tools enabling the description of the geodesics of a non-differentiable and fractal space–time geometry founded on the principle of the relativity of scales.

(3) Find the laws of coupling of scale and motion. In this case the scale variables become themselves functions of coordinates. These resulting scale fields, which are manifestations of the fractal geometry, can actually be identified to gauge fields of the electromagnetic, weak and strong interaction types, and are finally reincorporated in the motion equations.

At each of these steps, it is possible to suggest possible applications to biological systems through the definition of an effective "biological space–time" as a dynamic frame in which the biological objects are contained, followed by the identification of the trajectories of these objects as the geodesics of this biological space–time.

3. Construction and application of scale relativistic laws

3.1. Motivation

As we have seen in the statement of the general method of construction of the theory of scale relativity, the first step of this construction consists of finding the laws of explicit scale dependence which arise as a manifestation of the principle of scale relativity. In analogy with the physics of motion, we assume that these laws are solutions of differential equations (Nottale, 1993), but now acting also in scale space rather than only in classical space–time. In other words, we consider an infinitesimal dilation of the scale variable, i.e., an infinitesimal zoom, and we attempt to establish its effect on a physical quantity which is an explicit function of the resolution, in particular the fractal coordinates and the associated velocities themselves. We shall therefore be led to consider first order then second order differential equations of scale, which allow one to recover the usual self-similar fractals of constant fractal dimensions, but also to generalize them to a more general behaviour including scale symmetry breaking and variable fractal dimensions (Nottale, 1994, 1996, 1997, 2002).

In what follows, the variable to which we apply these methods is the length of a fractal curve that may also represent a coordinate in a fractal reference system. But all the results obtained may be easily generalized to fractal surfaces, fractal volumes, etc. The only difference between these cases is their topological dimension D_T. Since, as we shall see, it is not the fractal dimension D_F which directly appears in these equations, but a scale exponent $\tau = D_F - D_T$ (which will be identified to a "scale time"), the fractal dimension is simply $D_F = \tau$ for a set of points, $1 + \tau$ for lengths, $2 + \tau$ for surfaces, $3 + \tau$ for volumes (manifolds), etc. (Note that the case of fractal "volumes" such that $2 < D_F < 3$ constructed by iteratively removing smaller and smaller volumes from an initial three-dimensional object should be taken with caution, since their true topological dimension is $D_T = 2$).

Note also that, in order to simplify the presentation, we consider only in this section the dependence on scale, namely, we describe internal scale structures at a given "point" of standard space–time. However, it must be clear from the very beginning that this is only a first step of the approach, and that the functions we consider ultimately depend on scale variables, on positions (space) and on instants (time).

3.2. Scale invariance and Galilean scale relativity laws

3.2.1. Fractal coordinate and differential dilation operator

Consider a variable length measured on a fractal curve, and, more generally, a non-differentiable (fractal) curvilinear coordinate $\mathscr{L}(s, \varepsilon)$, that depends on some parameter s which characterizes the position on the curve and on the resolution ε. Such a coordinate generalizes to non-differentiable and fractal space–times the concept of curvilinear coordinates introduced for curved Riemannian space–times in Einstein's general relativity (Nottale, 1993).

As recalled in Section 1.3, such a scale-dependent fractal length $\mathscr{L}(s, \varepsilon)$, remains finite and differentiable when $\varepsilon \neq 0$, namely, one can define a slope for any resolution ε, being conscious that this slope is itself a scale-dependent fractal function. It is only at the limit $\varepsilon \to 0$ that the length is infinite and the slope undefined, i.e., that non-differentiability manifests itself.

Therefore the laws of dependence of this length upon position and scale may be written in terms of a double differential calculus, i.e., it can be the solution of differential equations involving the derivatives of $\mathscr{L}$ with respect to both s and ε.

As a preliminary step, we need to establish the relevant form of the scale variables and the way they intervene in scale differential equations. For this purpose, let us apply an infinitesimal dilation $\mathrm{d}\rho$ to the resolution, which is therefore transformed as $\varepsilon \to \varepsilon' = \varepsilon(1 + \mathrm{d}\rho)$. Being, at this stage, interested in pure scale laws, we omit the dependence on position in order to simplify the notation. By applying this transformation to a fractal coordinate $\mathscr{L}$, we obtain, to first order in the differential element,

$$\mathscr{L}(\varepsilon') = \mathscr{L}(\varepsilon + \varepsilon\,\mathrm{d}\rho) = \mathscr{L}(\varepsilon) + \frac{\partial \mathscr{L}(\varepsilon)}{\partial \varepsilon}\varepsilon\,\mathrm{d}\rho = (1 + \tilde{D}\,\mathrm{d}\rho)\mathscr{L}(\varepsilon), \tag{3}$$

where $\tilde{D}$ is, by definition, the dilation operator.

Recall that the concept of "operator" denotes a general way of describing a transformation applied on a quantity (scalar, vector, etc.). For example, it may be a simple product, $\hat{a} = a$, so that $\hat{a}f(x) = af(x)$ but also a differential operator, e.g., $\hat{a} = \partial/\partial x$, so that $\hat{a}f(x) = \partial f(x)/\partial x$, where ∂ denotes a partial derivative.

Since $\mathrm{d}\varepsilon/\varepsilon = \mathrm{d}\ln\varepsilon$, the identification of the two last members of Eq. (3) yields

$$\tilde{D} = \varepsilon\frac{\partial}{\partial \varepsilon} = \frac{\partial}{\partial \ln \varepsilon}. \tag{4}$$

This form of the infinitesimal dilation operator shows that the natural variable for the resolution is $\ln \varepsilon$, and that the expected new differential equations will indeed involve quantities such as $\partial\mathscr{L}(s,\varepsilon)/\partial \ln \varepsilon$. This theoretical result agrees and explains the current knowledge according to which most measurement devices (of light, sound, etc.), including their physiological counterparts (eye, ear, etc.) respond according to the logarithm of the intensity (e.g., magnitudes, decibels, etc.).

3.2.2. Self-similar fractals as solutions of a first order differential equation

Let us start by writing the simplest possible differential equation of scale, then by solving it. We shall subsequently verify that the solutions obtained comply with the principle of relativity. As we shall see, this very simple approach already yields a fundamental result: it gives a foundation and an understanding from first principles for self-similar fractal laws, which have been shown by Mandelbrot and many others to be a general description of a large number of natural phenomena, in particular biological ones (see e.g., Mandelbrot, 1975, 1982; Novak, 1999; Losa et al., 2002, other volumes of these series and references therein). In addition, the obtained laws, which combine fractal and scale-independent behaviours, are the equivalent for scales of what inertial laws are for motion. Since they serve as a fundamental basis of description for all the following theoretical constructions, we shall now describe their derivation in detail.

The simplest differential equation of explicit scale dependence which one can write is of first order and states that the variation of $\mathscr{L}$ under an infinitesimal scale transformation $\mathrm{d}\ln\varepsilon$ depends only on $\mathscr{L}$ itself. Basing ourselves on the previous derivation of the form of the dilation operator, we thus write

$$\frac{\partial \mathscr{L}(s,\varepsilon)}{\partial \ln \varepsilon} = \beta(\mathscr{L}). \tag{5}$$

The function β is *a priori* unknown. However, still looking for the simplest form of such an equation, we expand $\beta(\mathscr{L})$ in powers of $\mathscr{L}$, namely we write $\beta(\mathscr{L}) = a + b\mathscr{L} + \cdots$. Disregarding for the moment the s dependence, we obtain, to the first order, the following linear equation, in which a and b are constants:

$$\frac{\mathrm{d}\mathscr{L}}{\mathrm{d}\ln\varepsilon} = a + b\mathscr{L}. \tag{6}$$

In order to find the solution of this equation, let us change the names of the constants as $\tau = -b$ and $\mathscr{L}_0 = a/\tau$, so that $a + b\mathscr{L} = -\tau\,(\mathscr{L} - \mathscr{L}_0)$. We obtain the equation

$$\frac{\mathrm{d}\mathscr{L}}{\mathscr{L} - \mathscr{L}_0} = -\tau\,\mathrm{d}\ln\varepsilon, \tag{7}$$

i.e., $\mathrm{d}\ln(\mathscr{L} - \mathscr{L}_0) = -\tau\,\mathrm{d}\ln\varepsilon$. Its solution reads

$$\mathscr{L} = \mathscr{L}_0 + \mathscr{L}_1\varepsilon^{-\tau}, \tag{8}$$

where $\mathscr{L}_1$ is an integration constant. After a new redefinition of the constants ($\mathscr{L}_1 = \mathscr{L}_0\lambda^\tau$) it can be put under the form (see Fig. 3)

$$\mathscr{L}(\varepsilon) = \mathscr{L}_0\left\{1 + \left(\frac{\lambda}{\varepsilon}\right)^\tau\right\}. \tag{9}$$

This solution corresponds to a length measured on a fractal curve up to a given point. One can now generalize it to a variable length that also depends on the position characterized by the parameter s. One obtains

$$\mathscr{L}(s,\varepsilon) = \mathscr{L}_0(s)\left\{1 + \zeta(s)\left(\frac{\lambda}{\varepsilon}\right)^\tau\right\}, \tag{10}$$

in which, in the most general case, the exponent τ may itself be a variable depending on the position.

The same kind of result is obtained for the projections on a given axis of such a fractal length (Nottale, 1993). Let $X(s,\varepsilon)$ be one of these projections, it reads

$$X(s,\varepsilon) = x(s)\left\{1 + \zeta_x(s)\left(\frac{\lambda}{\varepsilon}\right)^\tau\right\}. \tag{11}$$

In this case $\zeta_x(s)$ becomes a highly fluctuating function which may be described by a stochastic variable, as can be seen in Fig. 4.

The important point here and for what follows is that the solution obtained is the sum of two terms, a classical (differentiable) part (that depends only on the position) and a fractal (non-differentiable) part (that depends on the position and on ε in a divergent way). By differentiating these two parts in the above

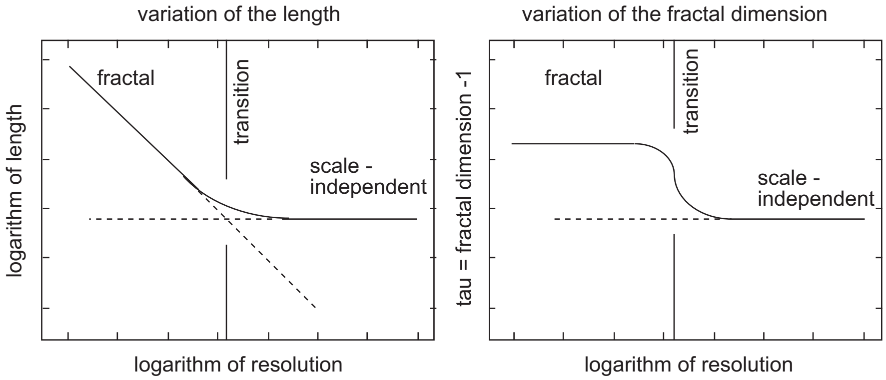

Fig. 3. Scale dependence of the length and of the effective fractal dimension D_F (or, equivalently, of the effective scale exponent or "scale time" $\tau = D_F - 1$), in the case of "inertial" scale laws (which are solutions of the simplest, first order scale differential equation): toward the small scale one gets a scale-invariant law with constant fractal dimension, while the explicit scale dependence is lost at scales larger than some transition scale, beyond which one recovers $D_F = D_T = 1$ (see text).

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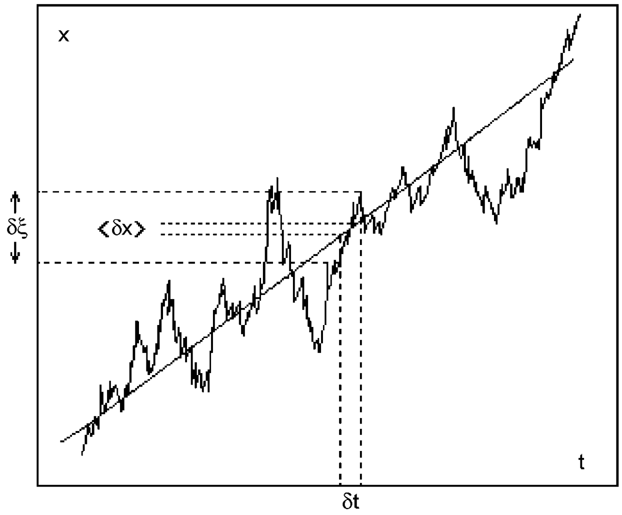

Fig. 4. A fractal function. An example of such a fractal function is given by the projections of a fractal curve on Cartesian coordinates, in function of a continuous and monotonous parameter (here the time t) which marks the position on the curve. The figure also exhibits the relation between space and time differential elements for such a fractal function, and compares the differentiable and non-differentiable parts of the space elementary displacement. While the "classical" coordinate variation $\delta x = \langle \delta X \rangle$ is of the same order as the time differential δt, the fractal fluctuation becomes much larger than δt when $\delta t \ll T$, where T is a transition time scale, and it depends on the fractal dimension D_F as: $\delta\xi \propto \delta t^{1/D_F}$. Therefore the two contributions to the full differential displacement are related by the fractal law $\delta\xi^{D_F} \propto \delta x$, since δx and δt are differential elements of the same order.

projection, we obtain the differential formulation of this essential result,

$$\mathrm{d}X = \mathrm{d}x + \mathrm{d}\xi, \tag{12}$$

where $\mathrm{d}x$ is a classical differential element, while $\mathrm{d}\xi$ is a differential element of fractional order (see Fig. 4, in which the parameter s that characterizes the position on the fractal curve has been taken to be the time t). This relation plays a fundamental role in the subsequent developments of the theory.

In the asymptotic small scale regime $\varepsilon \ll \lambda$, τ is constant (independent of scale), and one obtains a power-law dependence on resolution that reads

$$\mathscr{L}(s, \varepsilon) = \mathscr{L}_0(s)\left(\frac{\lambda}{\varepsilon}\right)^{\tau}. \tag{13}$$

We recognize in this expression the standard form of a self-similar fractal behaviour with constant fractal dimension $D_F = D_T + \tau$, which has already been found to yield a fair description of many physical and biological systems (Mandelbrot, 1982). Here the topological dimension is $D_T = 1$, since we deal with a length, but this can be easily generalized to surfaces ($D_T = 2$), volumes ($D_T = 3$), etc. The new feature here is that this result has been derived from a theoretical analysis based on first principles, instead of being postulated or deduced from a fit of observational data.

To be more specific, let us recall that there are two main ways to characterize a fractal curve (such as, e.g., the length of the British coast).

(i) One may measure it at various length scales, for example by varying the map scale. In this case, which is the one considered up to now, the resolution ε is a length interval, $\varepsilon = \delta X$, and one obtains the scale-dependent length given, by definition, by the above law $X(s, \delta X) = X_0(s) \times (\lambda/\delta X)^{D_F - 1}$. The exponent in this generic solution is then identified as $\tau = D_F - 1$.

Now, in the above solution, one may use time t as position parameter s, and if one travels on the curve at constant velocity, one obtains $X_0(t) = at$. Then a differential version of the above relation reads $\delta X = a\delta t(\lambda/\delta X)^{D_F - 1}$, so that we obtain the following fundamental relation between space elements and time elements on a fractal curve or function: $\delta X^{D_F} \propto \delta t$. Namely, they are differential elements of different orders, see Fig. 4.

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(ii) One may travel on the curve and measure its length on constant time intervals, then change the time scale. In this case the resolution ε is a time interval, $\varepsilon = \delta t$. One may therefore replace, in the above law (i) for the fractal length, the space resolution δX by its expression in function of the time resolution, $\delta X \propto \delta t^{1/D_F}$. Then the fractal length is now found to depend on the time resolution as $X(s, \delta t) = X_0(s) \times (T/\delta t)^{1-1/D_F}$. Therefore, in that case one may identify the exponent τ in the generic solution (13) with $\tau = 1 - 1/D_F$.

More generally, (in the usual case when $\varepsilon = \delta X$), following Mandelbrot, the scale exponent $\tau = D_F - D_T$ can be defined as the slope of the $(\ln \varepsilon, \ln \mathscr{L})$ curve, namely

$$\tau = \frac{\mathrm{d}\ \ln \mathscr{L}}{\mathrm{d}\ \ln(\lambda/\varepsilon)}. \tag{14}$$

For a self-similar fractal such as that described by the fractal part of the above solution, this definition yields a constant value which is the exponent in Eq. (13). However, one can anticipate on the following, and use this definition to compute an ‘‘effective’’ or ‘‘local’’ fractal dimension, now variable, from the complete solution that includes the transition to scale independence. Derivating the logarithm of Eq. (17) yields an effective exponent given by

$$\tau_{\mathrm{eff}} = \frac{\tau}{1 + (\varepsilon/\lambda)^{\tau}}. \tag{15}$$

This effective fractal dimension (minus 1) jumps from zero to its constant asymptotic value at the transition scale λ (see right part of Fig. 3).

3.2.3. *Galilean relativity of scales*

Let us now check that the fractal part of such a law is compatible with the principle of relativity extended to scale transformations of the resolutions (i.e., with the principle of scale relativity). It reads $\mathscr{L} = \mathscr{L}_0(\lambda/\varepsilon)^{\tau}$ (Eq. (13)), and it is therefore a law involving two variables ($\ln \mathscr{L}$ and τ) in function of one parameter (ε) which, according to the relativistic view, characterizes the state of scale of the system (its relativity is apparent in the fact that we need another scale λ to define it by their ratio). More generally, all the following statements remain true for the complete scale law including the transition to scale-independence, by making the replacement of $\mathscr{L}$ by $\mathscr{L} - \mathscr{L}_0$. Note that, to be complete, we anticipate on what follows and consider *a priori* τ to be a variable, even if, in the simple law first considered here, it remains a constant.

Let us take the logarithm of Eq. (13). It yields $\ln(\mathscr{L}/\mathscr{L}_0) = \tau \ln(\lambda/\varepsilon)$. The two quantities $\ln \mathscr{L}$ and τ then transform, under a finite scale transformation $\varepsilon \to \varepsilon' = \rho\varepsilon$, as

$$\ln \frac{\mathscr{L}(\varepsilon')}{\mathscr{L}_0} = \ln \frac{\mathscr{L}(\varepsilon)}{\mathscr{L}_0} - \tau \ln \rho, \tag{16}$$

and, to be complete,

$$\tau' = \tau. \tag{17}$$

These transformations have exactly the same mathematical structure as the Galilean group of motion transformation (applied here to scale rather than motion), which reads

$$X' = X - TV, \quad T' = T. \tag{18}$$

This is confirmed by the dilation composition law, $\varepsilon \to \varepsilon' \to \varepsilon''$, which writes

$$\ln \frac{\varepsilon''}{\varepsilon} = \ln \frac{\varepsilon'}{\varepsilon} + \ln \frac{\varepsilon''}{\varepsilon'}, \tag{19}$$

and is therefore similar to the law of composition of velocities between three reference systems K, K' and K'',

$$V''(K''/K) = V(K'/K) + V'(K''/K'). \tag{20}$$

Since the Galileo group of motion transformations is known to be the simplest group that implements the principle of relativity, the same is true for scale transformations.

It is important to realize that this is more than a simple analogy: the same physical problem is set in both cases, and is therefore solved under similar mathematical structures (since the logarithm transforms what

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would have been a multiplicative group into an additive group). Indeed, in both cases, it amounts to find the law of transformation of a position variable (X for motion in a Cartesian system of coordinates, $\ln \mathcal{L}$ for scales in a fractal system of coordinates) under a change of the state of the coordinate system (change of velocity V for motion and of resolution $\ln \rho$ for scale), knowing that these state variables are defined only in a relative way (V is the relative velocity between the reference systems K and K', and ρ is the relative scale: note that ε and ε' have indeed disappeared in the transformation law, only their ratio remains). This remark founds the status of resolutions as (relative) "scale velocities" and of the scale exponent τ as a "scale time".

Recall finally that, in physics, the Galileo group of motion is only a limiting case of the more general so-called Lorentz group discovered by Poincaré and Einstein.

3.2.4. Scale transition

We have found that the standard self-similar fractal laws can be derived from the scale relativity approach. However, it is important to note that Eq. (17), as shown in Fig. 3, gives something more. It also contains a breaking of the scale symmetry. Indeed, it is characterized by the existence of a transition from a fractal to a non-fractal behaviour at scales larger than some transition scale λ. In other words, contrarily to the case of motion laws, for which the invariance group is universal, the scale group symmetry is broken beyond some (relative) transition scale. The origin of this transition is, once again, to be found in relativity (namely, relativity of position and motion).

Indeed, if we start from a strictly scale-invariant law without any transition, $\mathcal{L} = \mathcal{L}_0(\lambda/\varepsilon)^{\tau}$, and add a translation in standard position space ($\mathcal{L} \to \mathcal{L} + \mathcal{L}_1$), we obtain

$$\mathcal{L}' = \mathcal{L}_1 + \mathcal{L}_0\left(\frac{\lambda}{\varepsilon}\right)^{\tau} = \mathcal{L}_1\left\{1 + \left(\frac{\lambda_1}{\varepsilon}\right)^{\tau}\right\}. \tag{21}$$

Therefore we recover the broken solution (that corresponds to the constant $a \neq 0$ in the initial scale differential equation). This solution is now asymptotically scale-dependent (in a scale-invariant way) only at small scales, and becomes independent of scale at large scales, beyond some transition λ_1 which is partly determined by the translation itself. This result is particularly relevant for applications to natural systems.

The scale symmetry is therefore spontaneously broken by the very existence of the standard space–time symmetries. The symmetry breaking is not achieved here by a suppression of one law to the profit of the other, but instead by a domination of each law (scale versus motion) over the other, respectively, toward the small and large scales. Since the transition is itself relative to the state of motion of the reference system, this implies that one can jump from one behaviour to the other by a change of the reference system. As we shall see in the companion paper, this transition plays an important role in the fractal space–time approach to quantum mechanics, since we identify it with the Einstein-de Broglie scale, and therefore the fractal to non-fractal transition with the quantum to classical transition (Nottale, 1993; Célérier and Nottale, 2004).

3.2.5. Multiple scale transitions

An important point concerning the scale symmetry, which is highly relevant to biological systems, is that, as is well-known from the observed scale-independence of physics at our own scales, the scale dependence is a spontaneously broken symmetry. In the previous section, we have shown that one very simply obtains one such symmetry breaking (either at upper scales or at lower scales) as a solution of the simplest possible scale differential equation. It turns out that multiple transitions can be obtained by a simple generalization (Nottale, 1997).

Let us still consider a perturbative approach and take the Taylor expansion of the differential equation $\mathrm{d}\mathcal{L}/\mathrm{d}\ln\varepsilon = \beta(\mathcal{L})$, but now to second order of the expansion. We obtain the equation:

$$\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}\ln\varepsilon} = a + b\mathcal{L} + c\mathcal{L}^2 + \cdots\ . \tag{22}$$

One of the solutions of this equation, which generalizes that of Eq. (9), describes a scaling behaviour which is broken toward both the small and large scales, as observed in most real fractal systems,

$$\mathcal{L} = \mathcal{L}_0\left(\frac{1 + (\lambda_0/\varepsilon)^{\tau}}{1 + (\lambda_1/\varepsilon)^{\tau}}\right). \tag{23}$$

Due to the non-linearity of the β function, there are now two characteristic scales in such a law. Indeed, as illustrated in Fig. 5,

*when $\varepsilon < \lambda_1 < \lambda_0$, one has $(\lambda_0/\varepsilon) \gg 1$ and $(\lambda_1/\varepsilon) \gg 1$, so that $\mathscr{L} = \mathscr{L}_0(\lambda_0/\lambda_1)^\tau \approx$ cst, independent of scale;

*when $\lambda_1 < \varepsilon < \lambda_0$, one has $(\lambda_0/\varepsilon) \gg 1$ but $(\lambda_1/\varepsilon) \ll 1$, so that the denominator disappears, and one recovers the previous pure scaling law $\mathscr{L} = \mathscr{L}_0\ (\lambda_0/\varepsilon)^\tau$;

*when $\lambda_1 < \lambda_0 < \varepsilon$, one has $(\lambda_0/\varepsilon) \ll 1$ and $(\lambda_1/\varepsilon) \ll 1$, so that $\mathscr{L} = \mathscr{L}_0 =$ cst, independent of scale.

3.2.6. *Scale relativity versus scale invariance*

Let us briefly be more specific about the way in which the scale relativity viewpoint differs from scaling or simple scale invariance. In the standard concept of scale invariance, one considers scale transformations of the coordinate,

$$X \to X' = q \times X, \tag{24}$$

then one looks for the effect of such a transformation on some function $f(X)$. It is scaling when

$$f(qX) = q^\alpha \times f(X). \tag{25}$$

The scale relativity approach involves a more profound level of description, since the coordinate X is now explicitly resolution-dependent, i.e., $X = X(\varepsilon)$. Therefore we now look for a scale transformation of the resolution,

$$\varepsilon \to \varepsilon' = \rho\varepsilon, \tag{26}$$

which implies a scale transformation of the position variable that takes, in the self-similar case, the form

$$X(\rho\varepsilon) = \rho^{-\tau} X(\varepsilon). \tag{27}$$

But now the scale factor on the variable has a physical meaning which goes beyond a trivial change of units. It corresponds to a coordinate measured at two different resolutions on a fractal curve of fractal dimension $D = 1 + \tau$, and one can obtain a scaling function of a fractal coordinate:

$$f(\rho^{-\tau} X) = \rho^{-\alpha\tau} \times f(X). \tag{28}$$

In other words, there are now three levels of transformation in the scale relativity framework (the resolution, the variable and its function) instead of only two in the usual conception of scaling (the variable and its function).

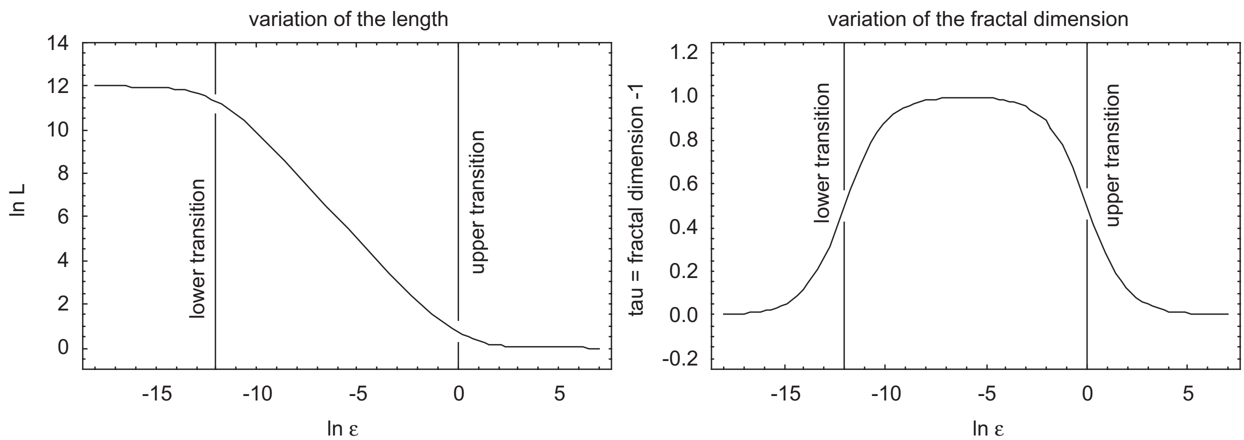

Fig. 5. Scale dependence of the length and of the "scale time" $\tau = D_F - 1$ (fractal dimension minus topological dimension) in the case of a self-similar behaviour (constant fractal dimension) involving two (upper and lower) scales of transition to scale independence (see text). This behaviour is a solution of a first order scale differential equation. Though the ratio of the upper and lower cut-offs is, in this example, as large as $e^{12} \approx 1.6 \times 10^5$, one finds that, due to the fact that the transitions occur on a finite range of scales (each of them cover a factor of about $e^4 \approx 50$), the range of scales over which the effective fractal dimension is constant remains small (≈ 50).

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3.3. Generalized scale laws

3.3.1. Discrete scale invariance, complex dimension and log-periodic behaviour

Fluctuations with respect to pure scale invariance are potentially important, namely the log-periodic correction to power laws that is provided, e.g., by complex exponents or complex fractal dimensions. It has been shown that such a behaviour provides a very satisfactory and possibly predictive model of the time evolution of many critical systems, including earthquakes and market crashes (Sornette, 1998 and references therein). More recently, it has been applied to the analysis of major event chronology of the evolutionary tree of life (Chaline et al., 1999; Nottale et al., 2000, 2002), of human development (Cash et al., 2002) and of the main economic crisis of western and precolumbian civilizations (Nottale et al., 2000; Johansen and Sornette, 2001).

Let us show how one can recover log-periodic corrections through the requirement of the scale covariance of scale differential equations (Nottale, 1997). Consider a scale-dependent function $\mathscr{L}(\varepsilon)$, (it may be for example the length measured along a fractal curve). In the applications to temporal evolution quoted above, the scale variable is identified with the time interval $|t - t_c|$, where t_c is the date of a crisis. Assume that $\mathscr{L}$ satisfies a first order differential equation,

$$\frac{d\mathscr{L}}{d \ln \varepsilon} - \nu\mathscr{L} = 0, \tag{29}$$

whose solution is a pure power law $\mathscr{L}(\varepsilon) \propto \varepsilon^\nu$ (cf. Section 3.2.2). Now looking for corrections to this law, we remark that simply incorporating a complex value of the exponent ν would lead to large log-periodic fluctuations rather than to a controllable correction to the power law. So let us assume that the right-hand side of Eq. (29) actually differs from zero

$$\frac{d\mathscr{L}}{d \ln \varepsilon} - \nu\mathscr{L} = \chi. \tag{30}$$

We can now apply the scale covariance principle and require that the new function χ be solution of an equation which keeps the same form as the initial equation

$$\frac{d\chi}{d \ln \varepsilon} - \nu'\chi = 0. \tag{31}$$

Setting $\nu' = \nu + \eta$, we find that $\mathscr{L}$ must be solution of a second-order equation

$$\frac{d^2\mathscr{L}}{(d \ln \varepsilon)^2} - (2\nu + \eta)\frac{d\mathscr{L}}{d \ln \varepsilon} + \nu(\nu + \eta)\mathscr{L} = 0. \tag{32}$$

The solution reads $\mathscr{L}(\varepsilon) = a\varepsilon^\nu(1 + b\varepsilon^\eta)$, and finally, the choice of an imaginary exponent $\eta = i\omega$ yields a solution whose real part includes a log-periodic correction:

$$\mathscr{L}(\varepsilon) = a\varepsilon^\nu[1 + b\cos(\omega \ln \varepsilon)]. \tag{33}$$

As previously recalled in Section 3.2.4, adding a constant term (a translation) provides a transition to scale independence at large scales (see Fig. 6).

3.3.2. Lagrangian approach to scale laws

In order to obtain physically relevant generalizations of the above simplest (scale-invariant) laws, the Lagrangian approach (briefly recalled in Section 2.2.1), can be used in scale space, leading to reverse the definition and meaning of the variables (Nottale, 1997, 2002).

This reversal is an analog to that achieved by Galileo concerning motion laws. Indeed, from the Aristotle's viewpoint, "time is the measure of motion". In the same way, the fractal dimension, in its standard (Mandelbrot's) acception, is defined from the topological measure of the fractal object (length of a curve, area of a surface, etc.) and resolution, namely (see Eq. (14))

$$t = x/v \leftrightarrow \tau = D_F - D_T = d \ln \mathscr{L}/d \ln(\lambda/\varepsilon). \tag{34}$$

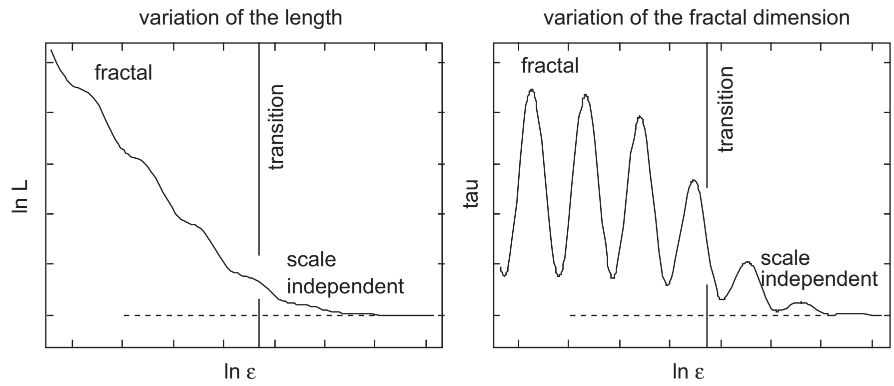

Fig. 6. Scale dependence of the length $\mathscr{L}$ of a fractal curve and of the effective "scale time" $\tau = D_F - 1$ in the case of a log-periodic behaviour with fractal/non-fractal transition at scale λ. The length reads in this case $\mathscr{L}(\varepsilon) = \mathscr{L}_0[1 + (\lambda/\varepsilon)^{\nu} e^{b\cos(\omega\ln(\varepsilon/\lambda))}]$, in terms of the scale variable ε.

In the case, mainly considered here, when $\mathscr{L}$ represents a length (i.e., more generally, a fractal coordinate), the topological dimension is $D_T = 1$ so that $\tau = D_F - 1$ (but, as recalled in the introduction of this section, it can be easily generalized to surfaces or volumes). With Galileo, time becomes a primary variable, and the velocity is deduced from space and time, which are therefore treated on the same footing, in terms of a space–time (even though the Galilean space–time remains degenerate because of the implicitly assumed infinite velocity of light).

In analogy, the scale exponent $\tau = D_F - 1$ becomes in this new representation a primary variable that plays, for scale laws, the same role as played by time in motion laws. We have suggested that this variable scale exponent be called "djinn" (meaning gift in Tibetan). (Note that it has been denoted as δ in previous papers, but we prefer here the less misleading notation τ, for scale time.)

Carrying on the analogy, in the same way as the velocity is the derivative of position with respect to time, $v = dx/dt$, we expect the derivative of position (in logarithm form $\ln \mathscr{L}$, since this is the relevant variable, as we have seen before) with respect to scale time τ to be a "scale velocity". Consider as reference the self-similar-like case, that reads $\ln \mathscr{L} = \tau \ln(\lambda/\varepsilon)$. Derivating with respect to τ, now considered as a variable, yields $d \ln \mathscr{L}/d\tau = \ln(\lambda/\varepsilon)$, i.e., the logarithm of resolution. By extension, we assume that this scale velocity provides a new general definition of resolution even in more general situations, namely,

$$\mathbb{V} = \ln\left(\frac{\lambda}{\varepsilon}\right) = \frac{d \ln \mathscr{L}}{d\tau}. \tag{35}$$

It is noticeable that, because the resolution ε is dimensioned, we are obliged to divide it by some reference scale λ when taking its logarithm: this is but a manifestation of the principle of scale relativity. Moreover, this new definition comes in support of the relativistic view of scale laws. Namely, in the same way as the velocity defines the relative state of motion of a system, the log of resolution, identified to a scale velocity, defines its relative state of scale.

A scale Lagrange function $\tilde{L}(\ln \mathscr{L}, \mathbb{V}, \tau)$ is introduced, from which a scale action is constructed

$$\tilde{S} = \int_{\tau_1}^{\tau_2} \tilde{L}(\ln \mathscr{L}, \mathbb{V}, \delta)\, d\tau. \tag{36}$$

The application of the action principle yields a scale Euler–Lagrange equation that writes

$$\frac{d}{d\tau}\frac{\partial \tilde{L}}{\partial \mathbb{V}} = \frac{\partial \tilde{L}}{\partial \ln \mathscr{L}}. \tag{37}$$

In analogy with the physics of motion, in the absence of any scale force (i.e., $\partial\tilde{L}/\partial \ln \mathscr{L} = 0$), the Euler–Lagrange equation becomes

$$\partial\tilde{L}/\partial\mathbb{V} = \text{const} \Rightarrow \mathbb{V} = \text{const}. \tag{38}$$

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which is the equivalent for scale of what inertia is for motion (Mandelbrot, 1982; Nottale, 1989, 1993). The simplest possible form for the Lagrange function is a quadratic dependence on the scale velocity, (i.e., $\tilde{L} \propto \mathbb{V}^2$). The constancy of $\mathbb{V} = \ln(\lambda/\varepsilon)$ means that it is independent of the scale time τ. Eq. (35) can therefore be integrated to give the usual power law behaviour, $\mathscr{L} = \mathscr{L}_0(\lambda/\varepsilon)^\tau$, i.e., we have recovered by this fundamental argument the expression of Eq. (13). This reversed viewpoint has several advantages which allow a full implementation of the principle of scale relativity:

(i) the scale time τ is given its actual status of a fifth dimension and the logarithm of the resolution, $\mathbb{V} = \ln(\lambda/\varepsilon)$, its status of scale velocity (see Eq. (35)). This is in accordance with its scale-relativistic definition, in which it characterizes the state of scale of the reference system, in the same way as the velocity $v = \mathrm{d}x/\mathrm{d}t$ characterizes its state of motion.

(ii) this allows one to generalize the formalism to the case of four independent space–time resolutions, $\mathbb{V}^\mu = \ln(\lambda^\mu/\varepsilon^\mu) = \mathrm{d}\ln\mathscr{L}^\mu/\mathrm{d}\tau$. This amounts to expand to a five-dimensional geometric description in terms of a space–time-djinn.

(iii) scale laws more general than the simplest self-similar ones can be derived from more general scale Lagrangians (Nottale, 1997) involving "scale accelerations" $\Gamma = \mathrm{d}^2\ln\mathscr{L}/\mathrm{d}\tau^2 = \mathrm{d}\ln(\lambda/\varepsilon)/\mathrm{d}\tau$, as we shall see in what follows.

Note however that there is also a shortcoming in this approach. Contrarily to the case of motion laws, in which time is always flowing toward the future (except possibly in elementary particle physics at very small time scales (Nottale, 1993), the variation of the scale time may be non-monotonic, as exemplified by the previous case of log-periodicity. Therefore this Lagrangian approach is restricted to monotonous variations of the fractal dimension, or, more generally, to scale intervals on which it varies in a monotonous way.

3.3.3. *Scale dynamics*

The previous discussion indicates that the scale invariant behaviour corresponds to freedom (i.e., scale force-free behaviour) in the framework of a scale physics. However, in the same way as there are forces in nature that imply departure from inertial, rectilinear uniform motion, we expect most natural fractal systems to also present distorsions in their scale behaviour with respect to pure scale invariance. This implies taking non-linearity in scale into account. Such geometric distorsions may be, as a first step, attributed to the effect of a dynamics of scale ("scale dynamics"), i.e., of a "scale field" (in analogy with the Newtonian interpretation of gravitation as the result of a force, which has later been understood from Einstein's general relativity theory as a manifestation of the curved geometry of space–time).

In this case the Lagrange scale-equation takes the form of Newton's equation of dynamics:

$$F = \mu \frac{\mathrm{d}^2 \ln \mathscr{L}}{\mathrm{d}\tau^2}, \tag{39}$$

where μ is a "scale mass", which measures how the system resists to the scale force, and where $\Gamma = \mathrm{d}^2\ln\mathscr{L}/\mathrm{d}\tau^2 = \mathrm{d}\ln(\lambda/\varepsilon)/\mathrm{d}\tau$ is the scale acceleration.

We shall now attempt to define in this framework physical, generic, scale-dynamical behaviours which could be common to very different systems (Nottale, 1997, 2002). For various systems the scale force may have very different origins, but in all cases where it has the same form (constant, harmonic oscillator, etc.), the same kind of scale behaviour would be obtained. It is also worthwhile to remark that such a Newtonian approach should be considered to be only an intermediate step toward a fully developed general scale relativity in which the scale forces can be expected to be finally recovered as approximations of the manifestations of the geometry of the scale space.

3.3.4. *Constant scale force*

Let us first consider the case of a constant scale force (with $\mu = 1$). The potential is, in this case, $\varphi = G\ln\mathscr{L}$, in analogy with the potential of a constant force f in space, which is $\varphi = -fx$, since the force is $-\partial\varphi/\partial x = f$. and Eq. (39) writes

$$\frac{\mathrm{d}^2 \ln \mathscr{L}}{\mathrm{d}\tau^2} = G. \tag{40}$$

This equation can be easily integrated. A first integration yields $\mathrm{d}\ln\mathscr{L}/\mathrm{d}\tau = G\tau + \mathbb{V}_0$, where $\mathbb{V}_0$ is a constant. Then a second integration yields a parabolic solution (which is the equivalent for scale laws of parabolic motion in a constant field):

$$\mathbb{V} = \mathbb{V}_0 + G\tau, \quad \ln\mathscr{L} = \ln\mathscr{L}_0 + \mathbb{V}_0\tau + \tfrac{1}{2}G\tau^2, \tag{41}$$

where $\mathbb{V} = \mathrm{d}\ln\mathscr{L}/\mathrm{d}\tau = \ln(\lambda/\varepsilon)$.

However the physical meaning of this result is not clear under this form. This is due to the fact that, while in the case of motion laws we search for the evolution of the system with time, in the case of scale laws we search for the dependence of the system on resolution, which is the directly measured observable. Since the reference scale λ is arbitrary, we can re-define the variables in such a way that $\mathbb{V}_0 = 0$, i.e., $\lambda = \lambda_0$. Indeed, from Eq. (41) we get $\tau = (\mathbb{V} - \mathbb{V}_0)/G = [\ln(\lambda/\varepsilon) - \ln(\lambda/\lambda_0)]/G = \ln(\lambda_0/\varepsilon)/G$. Then we obtain:

$$\tau = \frac{1}{G}\ln\left(\frac{\lambda_0}{\varepsilon}\right), \quad \ln\left(\frac{\mathscr{L}}{\mathscr{L}_0}\right) = \frac{1}{2G}\ln^2\left(\frac{\lambda}{\varepsilon}\right). \tag{42}$$

The scale time τ becomes a linear function of resolution (the same being true, as a consequence, of the fractal dimension $D_F = 1 + \tau$), and the $(\log\mathscr{L}, \log\varepsilon)$ relation is now parabolic instead of linear (see Fig. 7 and compare to Fig. 3). Note that, as in previous cases, we have considered here only the small scale asymptotic behaviour, and that we can once again easily generalize this result by including a transition to scale-independence at large scale. This is simply achieved by replacing $\mathscr{L}$ by $(\mathscr{L} - \mathscr{L}_0)$ in all the equations (see Fig. 7 and compare to Fig. 3).

There are several physical situations where, after careful examination of the data, the power-law models were clearly rejected since no constant slope could be defined in the $(\log\mathscr{L}, \log\varepsilon)$ plane. In the several cases where a clear curvature appears in this plane, e.g., turbulence (Dubrulle, 1997), sandpiles (Cafiero et al., 1995), fractured surfaces in solid mechanics (Carpinteri and Chiaia, 1996), the physics could come under such a scale-dynamical description. In these cases it might be of interest to identify and study the scale force responsible for the scale distorsion (i.e., for the deviation from standard scaling).

3.3.5. Scale harmonic oscillator

Another interesting case (Nottale, 1997) is that of a repulsive harmonic oscillator potential $\varphi = -(k/2)\ln^2\mathscr{L}$ in the scale space. The scale differential equation reads in this case (we omit the reference scale of $\mathscr{L}$ in order to simplify the description):

$$\frac{\mathrm{d}^2\ln\mathscr{L}}{\mathrm{d}\tau^2} = k\ln\mathscr{L}. \tag{43}$$

This equation is the scale equivalent of the well-known pendulum equation, $\mathrm{d}^2x/\mathrm{d}t^2 \pm \omega^2 x = 0$, whose solutions are sinusoidal in the attractive case and hyperbolic sinusoidal in the repulsive case. Setting $k = 1/\tau_0^2$,

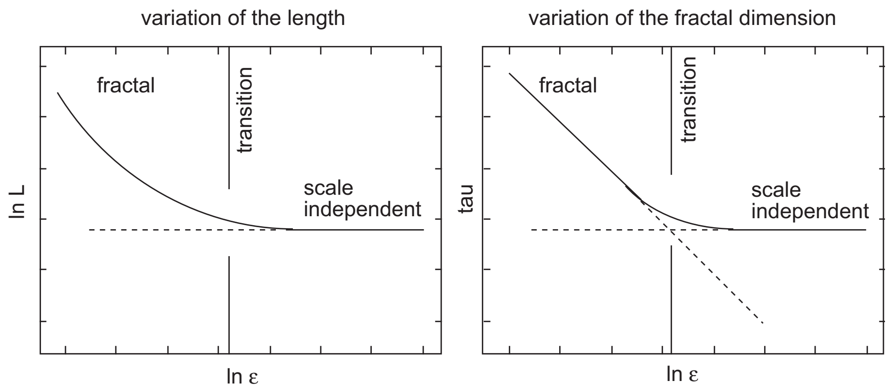

Fig. 7. Scale dependence of the length of a fractal curve $\ln\mathscr{L}$ and of its effective fractal dimension ($D_F = D_T + \tau$, where D_T is the topological dimension) in the case of a constant scale force, with an additional fractal/non-fractal transition.

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where τ_0 is constant, one of its solution reads

$$\ln \mathscr{L} = a \sinh\left(\frac{\tau}{\tau_0} + \alpha\right), \tag{44}$$

so that the scale velocity, which is its derivative with respect to τ, reads

$$\ln\left(\frac{\lambda}{\varepsilon}\right) = \frac{a}{\tau_0} \cosh\left(\frac{\tau}{\tau_0} + \alpha\right). \tag{45}$$

As in the previous section, we may now re-express $\ln \mathscr{L}$ in function of the resolution thanks to the relation $\cosh^2 x - \sinh^2 x = 1$, which yields

$$\frac{1}{\tau_0^2} \ln^2 \mathscr{L} - \ln^2\left(\frac{\lambda}{\varepsilon}\right) = -\frac{a^2}{\tau_0^2}. \tag{46}$$

Finally the solution can be put under the form (reintroducing a reference scale for $\mathscr{L}$ and changing the name of the constants)

$$\ln \frac{\mathscr{L}}{\mathscr{L}_0} = \tau_0 \sqrt{\ln^2\left(\frac{\lambda}{\varepsilon}\right) - \ln^2\left(\frac{\lambda}{\lambda_1}\right)}. \tag{47}$$

For $\varepsilon \ll \lambda$ it gives the standard Galilean case $\mathscr{L} = \mathscr{L}_0(\lambda/\varepsilon)^{\tau_0}$ (i.e., constant fractal dimension $D_F = 1 + \tau_0$). But its intermediate-scale behaviour is particularly interesting, since, from the viewpoint of the mathematical solution, resolutions larger than a scale λ_1 are no longer possible, as the term under the square root would become negative (this can clearly be only an approximation in reality). This transition of a new form therefore separates small scales from large scales, i.e., an "interior" (scales smaller than λ_1) from an "exterior" (scales larger than λ_1). It is characterized by an effective fractal dimension that becomes formally infinite.

Here λ is the fractal/non-fractal transition scale for the asymptotic domain, i.e., it is the transition scale which would have been observed in the absence of the additional scale force (see Fig. 8).

Another possible interpretation of this scale harmonic oscillator model is to consider the variable ε in the above equations as a distance r from a centre (i.e., a scaling coordinate instead of a scale resolution). Then it describes a system in which the effective fractal dimension of trajectories diverges at some distance $r = \lambda_{max}$ from the centre, is larger than 1 in the inner region and becomes 1 (i.e., non-fractal) in the outer region. Since the increase of the fractal dimension of a curve corresponds to the increase of its "thickness" (see Nottale, 1993, p. 80 for an example of a fractal curve whose fractal dimension varies with position), such a model can be interpreted as describing a system in which the inner and outer domains are separated by a wall (Fig. 8).

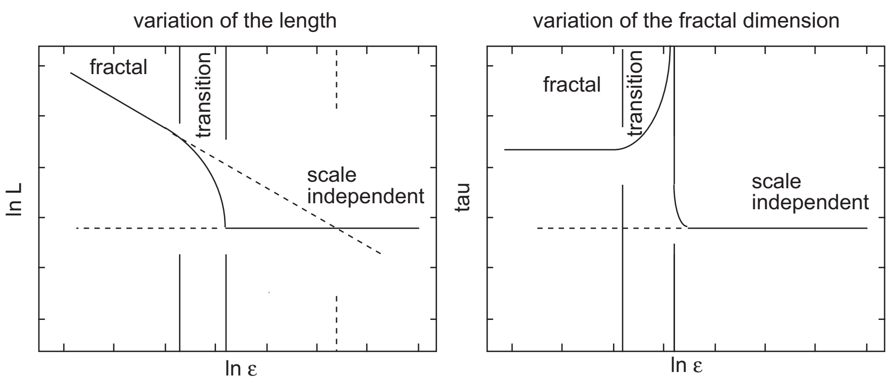

Fig. 8. Scale dependence of the length of a fractal curve and of its effective fractal dimension (minus topological dimension) in the case of a harmonic oscillator scale potential.

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3.4. Application to biological systems

3.4.1. Species evolution and embryogenesis

Let us first consider the log-periodic generalization to scale invariance obtained in Section 3.3.1. Several studies have shown that many biological, natural, sociological and economic phenomena obey a log-periodic law such as can be found in several critical phenomena: earthquakes (Sornette and Sammis, 1995), stock market crashes (Sornette et al., 1996), evolutionary leaps (Chaline et al., 1999; Nottale et al., 2000, 2002), long time scale evolution of western and other civilizations (Nottale et al., 2000, 2002), world economy indices dynamics (Johansen and Sornette, 2001), embryogenesis (Cash et al., 2002), etc. Thus emerges the idea that this behaviour typical of temporal crisis could be extremely widespread, as much in the organic world as in the inorganic one (Sornette, 1998).

In the case of species evolution, one observes the occurrence of major evolutionary leaps leading to bifurcations among species. The global pattern is assimilated to the tree of life, whose bifurcations are identified to evolutionary leaps, and branch lengths to the time intervals between these major events (Chaline et al., 1999). Such a reasoning leads to suggest a log-periodic probability law for these events which would be a solution of a second order scale-covariant differential equation (see Section 3.3.1). In this case the dates of the probability peaks are given by a law of the type $T_n = T_c + (T_0 - T_c)g^{-n}$, where T_c is the critical convergence time (or critical divergence time in the case of deceleration), T_0 is any event in the lineage which is used as reference, n the rank of occurrence of a given event and g is the scale ratio between successive time intervals. Such a chronology is periodic in terms of logarithmic variables, i.e., $\log(T_n - T_c) = \log(T_0 - T_c) - n \log g$.

The application of this model to the fossil record data has given positive results. Indeed, a statistically significant log-periodic acceleration has been found at various scales for global life evolution, for primates, for sauropod and theropod dinosaurs, for rodents and North American equids. A deceleration law was conversely found in a statistically significant way for echinoderms and for the first steps of rodents evolution (see Fig. 9 and more details in Chaline et al., 1999; Nottale et al., 2000, 2002). One finds either an acceleration toward a critical date T_c or a deceleration from a critical date, depending on the considered lineage.

Considering the relationships between phylogeny and ontogeny, it appeared interesting to verify whether the log-periodic law describing critical biological, inorganic and economic phenomena may also be applied to the various stages in human embryological development. The result, shown in Fig. 10, is that a statistically significant log-periodic deceleration is indeed observed, starting from a critical date that is nothing but the conception date (Cash et al., 2002).

These results are not mere analogies. Indeed, although they remain, at that stage, of an empirical nature (it is only a purely chronological analysis which does not take into account the nature of the events), they nevertheless allow predictivity. Indeed, the fitting law is a two-parameter function (T_c and g) that is applied to time intervals, so that only three events are needed to define these parameters. Therefore the subsequent dates are predicted, in a statistical way, since they are interpreted as the dates of the peaks of probability for an event to happen. As expected, the observed dates (points in Fig. 9) show small fluctuations with respect to the predicted dates (dashes in Fig. 9).

Examples of such a predictivity (or retropredictivity) are: (i) the retroprediction that the common Homo-Pan-Gorilla ancestor (expected e.g., from genetic distances and phylogenetic studies), has a more probable date of appearance at ≈ -10 million years (Chaline et al., 1999); its fossil has not yet been discovered; (ii) the prediction of a critical date for the long term evolution of human societies around the years 2050–2080 (Nottale et al., 2000, 2002; Johansen and Sornette, 2001); (iii) the finding that the critical dates of rodents may reach $+60$ Myrs in the future, showing their large capacity of evolution, comforted by their known high biodiversity; (iv) the finding that the critical dates of dinosaurs are about -150 Myrs in the past, indicating that they had reached the end of their capacity of evolution (at least for the morphological characters studied) well before their extinction at -65 Myrs; (v) the finding that the critical date of echinoderms (which decelerate instead of accelerating) is, within uncertainties, the same as that of the PreCambrian–Cambrian radiation, this supporting the view of the subsequent events as a kind of "scale wave" expanding from this first shock.

As concerns the nature of the events themselves, some statements may be made about them in the context of a more sophisticated version of the scale relativity theory (see companion paper).

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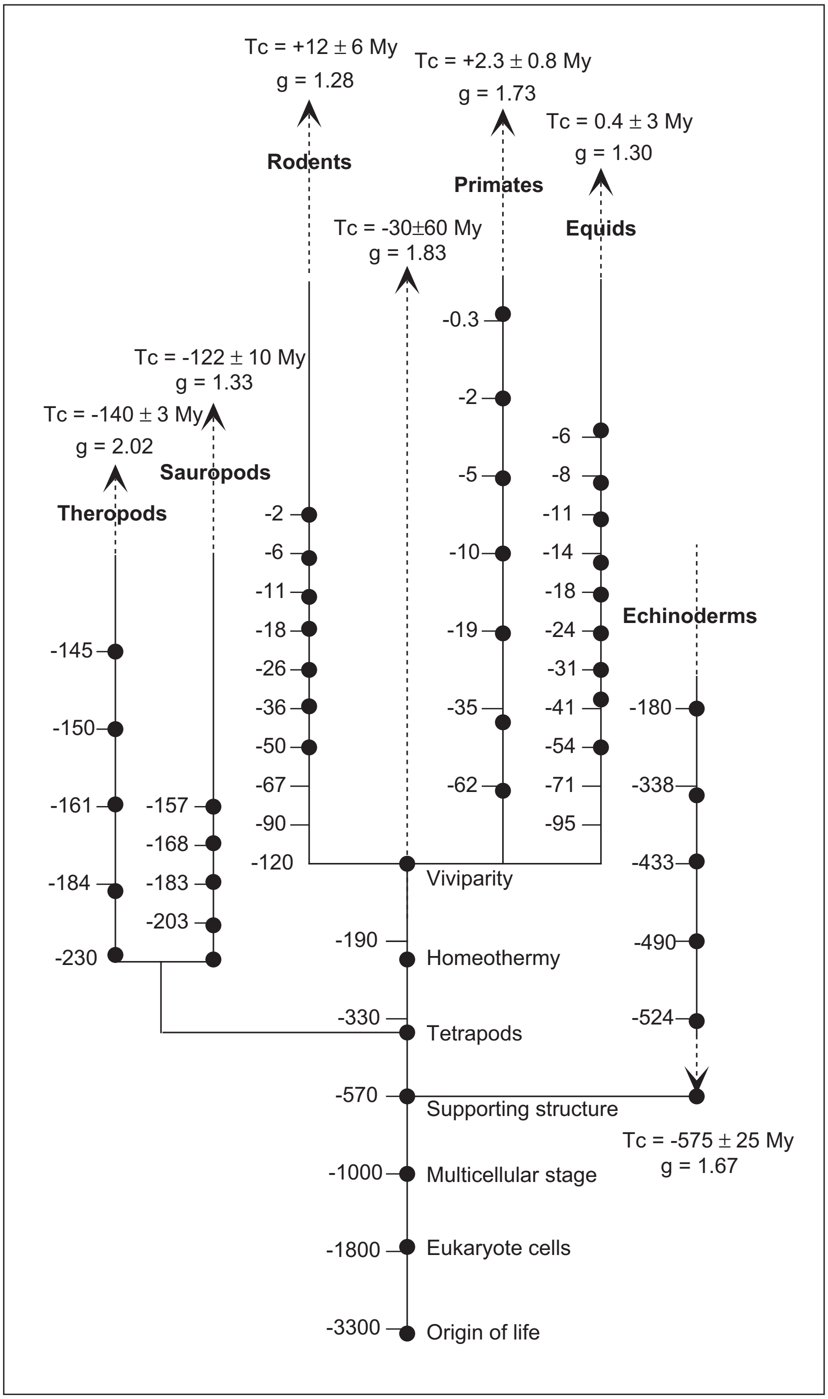

Fig. 9. The dates of major evolutionary events of seven lineages (common evolution from life origin to viviparity, Theropod and Sauropod dinosaurs, Rodents, Equidae, Primates including Hominidae, and Echinoderms) are plotted as black points in terms of $\log(T_c - T)$, and compared with the numerical values from their corresponding log-periodic models (computed with their best-fit parameters). The adjusted critical time T_c and scale ratio g are indicated for each lineage (Chaline et al., 1999; Nottale et al., 2000, 2002).

3.4.2. Confinement

The solutions of non-linear scale equations such as that involving a harmonic oscillator-like scale force of Section 3.3.5 may also be meaningful for biological systems. Indeed, its main feature is its capacity to describe

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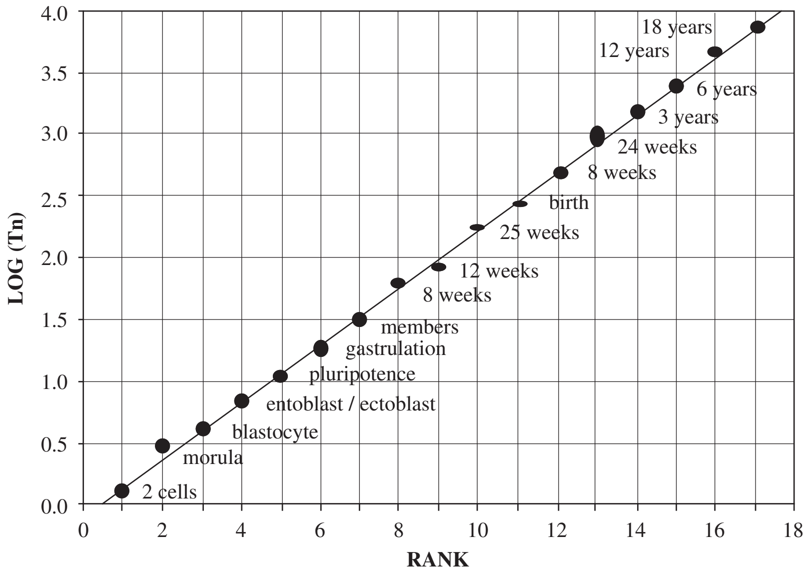

Fig. 10. Decimal logarithm $\log(T_n)$ of the dates of the steps of human development, counted starting from the conception date, plotted in terms of their rank n, showing a log-periodic deceleration with a scale ratio $g = 1.71 \pm 0.01$ (Cash et al., 2002). The vertical size of the points gives the confidence interval for the dates (an error bar of ±12% on the first three dates has been assumed).

a system in which has emerged a clear separation between an inner and an outer region, which is one of the properties of the first prokaryotic cell.

Consider indeed the first interpretation of the solution where ε is a varying scale of resolution (e.g., one considers the multiple versions of the system obtained by covering it with balls of varying radii ε). The deformation obtained due to the additional scale force resembles what would be obtained, for example, by compacting a sponge. Namely, its largest scale is decreased (this is represented by the decrease of the transition scale from λ to $\lambda_{\max}$, see left part of Fig. 8). Now, when one looks at smaller and smaller scales in the material (given, e.g., by the radii of the holes in the sponge), one finds that they become closer and closer to the unaffected self-similar case (namely, the smaller holes in the sponge are not affected by the overall diminution of size). Therefore the effect of the scale harmonic oscillator force results in a confinement of the large scale material in such a way that the small scales remain unaffected.

Remark also that all the solutions obtained have also a reversed counterpart, in which the fractality is toward the large scales and the transition toward the small scales. In this case it would be the large scales of the system which would remain unaffected against a perturbation having increased the small scales. Such behaviour could also be of technological and biological interest (it corresponds to the role of a bumper).

Concerning the second interpretation of these laws, in which the zone of diverging fractal dimension is at a given distance from the centre of a system, it can be interpreted as the description of a membrane. It is indeed the very nature of biological systems to have not only a well-defined size and a well-defined separation between interior and exterior, but also systematically an interface between them, such as membranes or walls. This is already true of the simplest prokaryote living cells. Therefore this result suggests the possibility that there could exist a connection between the existence of a scale field (e.g., a global pulsation of the system) both with confinement of the cellular material and with the appearance of a limiting membrane or wall (Nottale, 2004). This is reminiscent of eukaryotic cellular division which involves both a dissolution of the nucleus membrane and a deconfinement of the nucleus material, transforming before the division an eukaryote into a prokaryote-like cell. Once again, this is not a mere analogy, since this could be a key toward a better understanding of the first major evolutionary leap after the appearance of cells, namely the emergence of eukaryotes, a proposal which we intend to test through effective future experiments.

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4. Conclusion to part 1

At this stage, it is already apparent from the examples discussed in the previous section that the scale relativity theory framework offers the opportunity to establish elements of the theories of evolutionary, developmental and cell biology on the same first principles, thus potentially overcoming the fundamental hurdles of multiscale integration in systems biology. This will be further discussed in the companion paper, in which we will also explore the prospects offered by the scale-relativistic framework for the development of generalized quantum-type scale laws, and a tentative extension to scale space suggesting novel types of experiments and devices enabling trans-disciplinary systems biology programmes focused on the analysis, engineering and management of physical and biological systems with macroscopic quantum-type behaviours.

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Appendix A. Structure of field theories

Classical field theories are characterized by a general structure that is preserved whatever the level of complexity of these theories. We shall briefly exemplify this fundamental point with the three archetypes for a scalar (Newton's universal theory of gravitation), a vectorial (Maxwell's theory of electromagnetism), then a tensorial theory (Einstein's relativistic theory of gravitation).

Namely, the theory relies on the definition of a potential (or equivalently of a potential energy), of which the field derives. Then one defines charges for this field. The field equations state that the gradient of the field is given by the current of the charges. The equation of motion of the particles in the field have the form of Newton's equation of dynamics in terms of a force which is a function of the field and the velocity of the particle.

In Newton's theory, the potential is scalar and therefore the field, identical with the force, is vectorial. The charges are the masses:

$$\text{Potential energy } \phi \rightarrow \text{field (force) } \mathbf{F} = -\nabla\phi,$$
$$\text{Charges } m,\ \text{density } \rho \rightarrow \text{current } J = 4\pi G\rho m,$$
$$\text{Field equations } \nabla.\mathbf{F} = -J,$$
$$\text{Motion equations } m\frac{\mathrm{d}^2\mathbf{x}}{\mathrm{d}t^2} = \mathbf{F}.$$

In Maxwell's theory, the potential becomes vectorial, so that the field is a tensor. The charges remain scalar quantities (the electric charges), and the current is described by the velocity of the charges, $q\mathbf{u}$, and it is therefore vectorial. The force involves the field and the particle velocity.

Finally, in Einstein's theory, the potential is now a two-index tensor, so that the field is a three-index object called the Christoffel symbol. The charges become vectorial since it is no longer only the masses that create gravitational fields (i.e., in Einstein's theory, it is the curvature of space–time), but the full momentum energy content of the Universe. Therefore the current becomes a tensorial quantity, the energy-momentum tensor.

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