

Notes on designing class-E RF power amplifiers

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Abstract

Perhaps the most difficult part of any design process is the synthesis of suitable candidate circuits based on the results of an analytical model. This work focusses on a simplified analytical model for the class-E switch-mode amplifier using an inductive choke in the bias circuit. For microwave uses, a $1/4$ wave choke is useful, but requires a somewhat more complicated analysis (to be added later).

1 Introduction

In battery powered, space applications, and increasingly, fixed mains powered RF systems, power efficiency has become an important design requirement. In transmitting equipment, the RF final power amplifier is often the largest consumer of power and the place where gains in overall system power efficiency can be most effective.

By treating the power amplifier active device more as a switch than a voltage-variable resistor, efficiency gains can be readily realised. This paper will focus on the interesting case of the class-E switching power amplifier. The main points covered can be summarised as:

- Description of inductor choke biased amplifier,
- Analysis of the amplifier,
- Choice of critical components based on analysis,
- Verification of design using SPICE model,

The analysis will follow that presented in Grebennikov and Sokal [1].

2 The inductive choke drain bias

2.1 Description

The conceptual form of the class-E amplifier is deceptively simple. Figure 1 shows the circuit in its basic form with the series inductors displayed in “two parts”, whose reason will become clear later on.

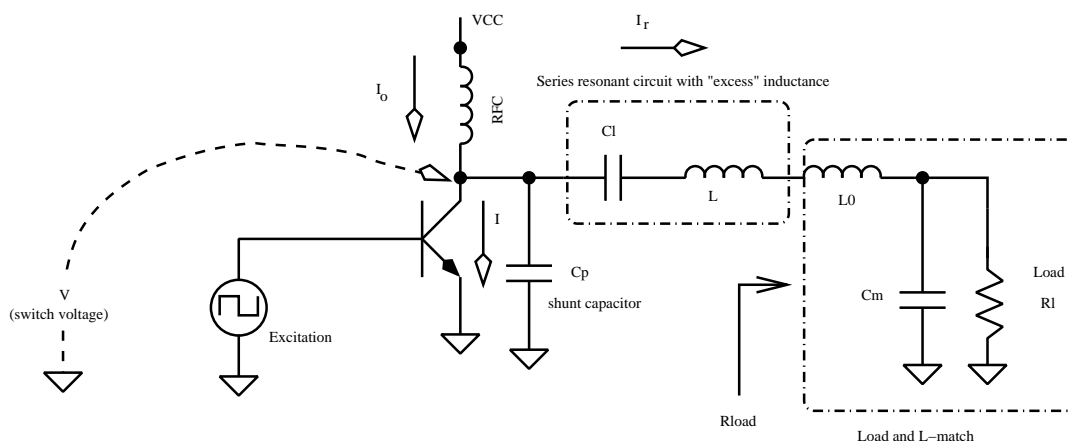


Figure 1: Basic schematic of Class-E amplifier.

The transistor is driven such that it spends a most of its time either in a completely off state or a completely on state. Put another way, there is little overlap in the switch voltage and current waveforms. Where the switch voltage and current waveforms overlap is where power losses are incurred in the switch. Of course, there are losses from parasitic resistance in inductors and capacitors as well, but we will ignore these in this analysis.

The class-E amplifier has two important characteristics, namely the presence of the shunt capacitor across the switch and a net series load inductance that gives the required phase shift for the fundamental wave and behaves as a harmonic open circuit.

2.2 Analysis

In order to simplify the analysis, we assume that

- the switch is “perfect” (i.e., instantaneous opening and closing and lossless),
- the RF choke on the DC supply looks like an ideal open circuit at the frequency of operation,

- the switch duty cycle is (an optimal) 50%,
- the Q of the load RLC circuit is high enough that the passage of harmonic signal to the load is negligible,
- and that the Class-E conditions are satisfied when the switch closes at $\omega t = 2\pi n$:

$$V(\omega t)|_{\omega t=2\pi n} = 0 \quad (1)$$

$$\frac{d}{d(\omega t)}V(\omega t)\Big|_{\omega t=2\pi n} = 0. \quad (2)$$

The voltage across the switch is given by $V(\omega t)$ and the current through the switch is denoted by $I(\omega t)$ at any instant in normalised time ωt . For convenience, a table of variables relevant to the analysis is given below.

Table 1: Table of pertinent variables.

Variable description	Variable	Notes
DC supply voltage	V_{cc}	
DC supply current	I_0	
Load current	$I_r(\omega t)$	single sine wave
Shunt capacitor current	$I_c(\omega t)$	only when switch is open
Switch current	$I(\omega t)$	Only when switch is closed
Shunt capacitor value	C_p	
Series capacitor value	C_l	
Series inductor value	L	$L = L_{res} + L_{ext}$
Series resonant inductor	L_{res}	
Series loading inductor	L_{ext}	extra inductance for Class-E
L-Match inductor	L_0	transform load impedance
L-match capacitor	C_m	
Load resistance	R_l	usually 50 Ω
Load resistance	R	before L-match; a low value

Using the results of 2, the current through the capacitor immediately after the switch closes must be zero, i.e.

$$I_c(\omega t) = \omega C \frac{d}{d(\omega t)}V(\omega t) = 0. \quad (3)$$

This provides us with the initial condition for the switch current for the time $2\pi n < \omega t < (2n + 1)\pi$:

$$I = I_0 + I_r \quad (4)$$

at the moment of turn-on. If $I_r(\omega t) = I_{r0} \sin(\omega t + \phi)$, we get

$$I_0 = -I_{r0} \sin \phi, \quad (5)$$

at $\omega t = 2\pi n$, where ϕ is a yet-to-be-determined phase angle. This allows us to write the switch current during its open period (during $(2n + 1)\pi < \omega t < 2n + 2)\pi$) as

$$I(\omega t) = I_0 + I_{r0} \sin(\omega t + \phi) \quad (6)$$

or, on substitution from equation (4),

$$I(\omega t) = I_{r0} (\sin(\omega t + \phi) - \sin \phi). \quad (7)$$

The voltage across the shunt capacitor can be written now as

$$V(\omega t) = -\frac{1}{\omega C} \int_{\pi}^{\omega t} I(\omega t) d(\omega t) = -\frac{I_{r0}}{\omega C} (\cos(\omega t + \phi) + \cos \phi + (\omega t - \pi) \sin \phi) \quad (8)$$

At $\omega t = 2\pi n$ (or with no loss in generality $\omega t = 0$), the first class-E condition ($V(0) = 0$) yields

$$2 \cos \phi + \pi \sin \phi = 0 \quad (9)$$

or, equivalently

$$\tan \phi = -\frac{2}{\pi}. \quad (10)$$

This is the requirement for the fundamental phase shift that permits class-E operation. The current will lag the voltage waveform by $\phi = -32.48^\circ$.

Two relationships that will be useful are

$$\sin \phi = \frac{-2}{\sqrt{\pi^2 + 4}}, \quad (11)$$

$$\cos \phi = \frac{\pi}{\sqrt{\pi^2 + 4}}. \quad (12)$$

Rewriting (8) using these trigonometry relationships yields

$$V(\omega t) = -\frac{I_{r0}}{\omega C} (\pi \cos \omega t + 2 \sin \omega t + \pi - 2(\omega t - \pi)) \frac{1}{\sqrt{\pi^2 + 4}}. \quad (13)$$

If we use (5) and substitute the trig identity to get $I_0 = 2I_{r0}/\sqrt{\pi^2 + 4}$, we get the final form of the voltage across the switch when it is open:

$$V(\omega t) = \frac{I_0}{\omega C} \left(\omega t - \frac{3\pi}{2} - \frac{\pi}{2} \cos \omega t - \sin \omega t \right) \quad (14)$$

for $\pi \leq \omega \leq 2\pi$ (removing the period index n).

Since the choke inductor is assumed to be ideal (no resistance) the supply voltage V_{cc} must be equivalent to the average voltage appearing across the switch/shunt capacitor over a full on-off cycle. This produces the first important design relationship

$$V_{cc} = \frac{1}{2\pi} \int_{\pi}^{2\pi} V(\omega t) d(\omega t) = \frac{I_0}{\pi\omega C}. \quad (15)$$

Since the switch is defined to be lossless and the class-E conditions are satisfied, the DC power provided by the bias DC source should be equal to the power dissipated in the load resistor. Defining the resistance to be that *before* the L-match to be R , we have

$$I_0 V_{cc} = \frac{1}{2} I_{r0}^2 R \quad (16)$$

By substituting $I_0 = 2I_{r0}/\sqrt{\pi^2 + 4}$ into (16), we get

$$\frac{2I_{r0}}{\sqrt{\pi^2 + 4}} V_{cc} = \frac{1}{2} I_{r0}^2 R. \quad (17)$$

Rearranging yields

$$\frac{4V_{cc}}{\sqrt{\pi^2 + 4}} = I_{r0} R = V_{r0} \quad (18)$$

and for I_0 :

$$I_0 = \frac{V_{r0}}{R} \sin \phi = \frac{8V_{cc}}{(\pi^2 + 4) R} \quad (19)$$

Hence, the power delivered to the load is

$$P_{load} = \frac{8V_{cc}^2}{(\pi^2 + 4) R} \quad (20)$$

Based on the calculations so far, we can specify the amplifier power output in terms of the V_{cc} and load resistance. We can also specify I_0 and hence shunt capacitance C . Note that the shunt capacitance C contains the parasitic collector (or drain) capacitance of the switch. Hence, this will affect the choice of switching transistor. The design process involves finding a good combination of V_{cc} , R , C that allows for a realisable amplifier. Generally R should not be too small otherwise the series inductor will be too small to be physically realisable.

At this point, we derive the required value for the extra inductance L_{ext} . Since we have an expression for the fundamental frequency current through the load, we can find the voltage across the load resistor and L_{ext} from

$$V_R = \frac{1}{\pi} \int_{\pi}^{2\pi} V(\omega t) \sin(\omega t + \phi) d(\omega t) \quad (21)$$

$$V_{L_{ext}} = \frac{1}{\pi} \int_{\pi}^{2\pi} V(\omega t) \cos(\omega t + \phi) d(\omega t). \quad (22)$$

Applying Kirchoff's voltage law, we see that

$$\frac{V_{L_{ext}}}{V_R} = \frac{\omega L_{ext}}{R} = \frac{\pi + 2 \sin 2\phi - (\pi/2) \cos 2\phi}{(\pi/2) \sin 2\phi + 2 \cos 2\phi}. \quad (23)$$

The excess phase $\phi = -32.43^\circ$, so the preceding expression simplifies to

$$\frac{\omega L_{ext}}{R} = 1.153. \quad (24)$$

The next step is to treat the series resonant part of the output circuit, L_{res} and C_l . The center frequency and load resistance R are already specified in the design. What remains is to specify the Q-factor of the resonant circuit. Any value above 5-7 should suffice. Lower Qs will require "some tweaking" of the excess inductance and shunt capacitor values as harmonic leakage becomes significant. Low-Q permits operation over wider bandwidths, but at the cost of more harmonic leakage (unless a higher-order output filter network is used). From a straightforward analysis of a series resonant LCR network we can find the values of resonant capacitance and inductance:

$$C_l = \frac{1}{\omega QR} \quad (25)$$

$$L_{res} = \frac{QR}{\omega} \quad (26)$$

The final part of this basic design is the specification of the L-match network. Usually the required load resistance for the Class-e amplifier is of the order of a few ohms. This needs to be transformed up to a standard load impedance; usually 50 or 75 ohms. The L-match is a convenient way to do this (as well as providing some extra harmonic rejection as a side-effect). If the final load resistor is defined as R_l , then it is a simple matter to calculate the required L_0 and C_m from

$$C_m = \frac{1}{\omega R_l} \sqrt{\frac{R_l}{R} - 1} \quad (27)$$

$$L_0 = C_m R R_l \quad (28)$$

The final value for the series inductor corresponds to the sum of all the computed inductances:

$$L_{total} = L_{ext} + L_{res} + L_0. \quad (29)$$

We now have everything we need to proceed with a practical design.

2.3 A practical example output network

As an example, let us define an amplifier with the following specifications.

Table 2: Example amplifier specification

V_{cc}	12V
P_{load}	5W
ω	$2\pi 6.9\text{MHz}$
Q	10
R_l	50Ω

The current drawn from the DC supply will be

$$I_0 = \frac{P_{load}}{V_{cc}} = \frac{5}{12} = 0.4167A. \quad (30)$$

Hence, we need a shunt capacitance of

$$C = \frac{I_0}{\omega \pi V_{cc}} = \frac{0.4167}{(6.283)(6.9e6)(3.1415)(12)} = 255pF. \quad (31)$$

This means we need a transistor with a parasitic collector/drain capacitance less than this value.

The required load resistance seen by the switch is computed using

$$R = \frac{8V_{cc}^2}{(\pi^2 + 4)P_{load}} = \frac{(8)(144)}{(13.869)(5)} = 16.6\Omega \quad (32)$$

The excess series inductance L_{ext} is found from

$$L_{ext} = \frac{(1.153)R}{\omega} = \frac{(1.153)(16.6)}{(6.283)(6.9e6)} = 441nH. \quad (33)$$

At this frequency, this value of inductance is easily realisable.

Next, we move on to computing the resonant circuit parameters. Based on the specified Q of 10, we get

$$C_l = \frac{1}{QR\omega} = \frac{1}{(10)(16.6)(6.283)(6.9e6)} = 139pF \quad (34)$$

$$L_{res} = \frac{QR}{\omega} = \frac{(10)(16.6)}{(6.283)(6.9e6)} = 3.829uH \quad (35)$$

Finally, we design the L-match using

$$C_m = \frac{1}{\omega R_l} \sqrt{\frac{R_l}{R} - 1} = \frac{\sqrt{(50)/(16.6) - 1}}{(6.283)(6.9e6)(50)} = 652pF \quad (36)$$

$$(37)$$

$$L_0 = RR_l C_m = (16.6)(50)(6.52e - 10) = 544nH \quad (38)$$

The value of the series inductor is just the sum of all the computed inductances: 4.81uH. The full circuit is in Figure 2.

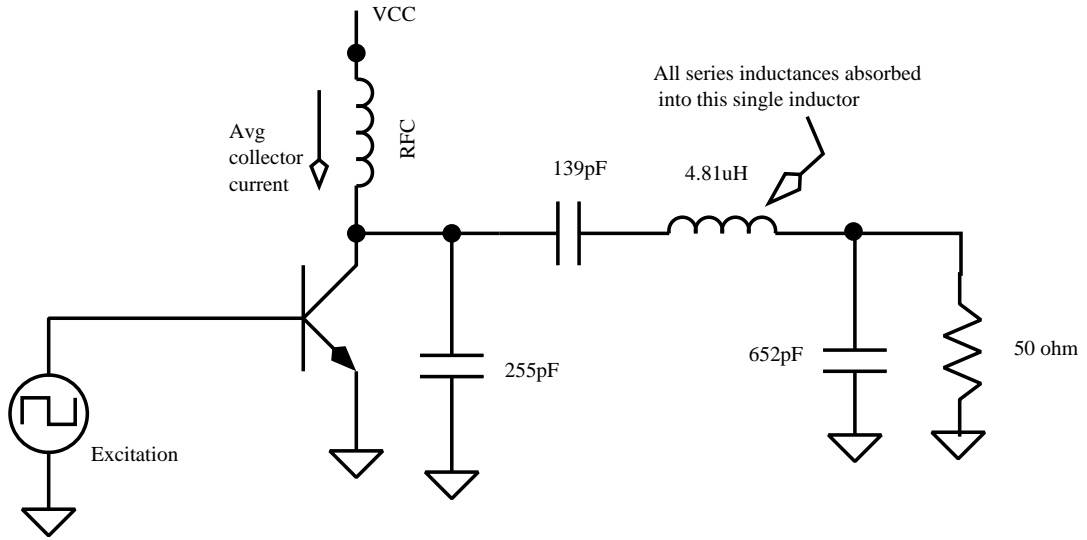


Figure 2: Schematic of 6.9MHz Class-E amplifier.

The value of the RF choke inductor is not specified. In practical terms, its specific value is not all that critical, as long as its impedance is at least an order of magnitude higher than the load resistance and it is not self-resonant at the first three or four harmonics (it needs to look like an open circuit to these harmonics, if possible).

2.4 SPICE verification of design

The choice of components was verified using a time domain SPICE simulation of the Class-E output network fed by a (nearly) ideal switch. Class-E operation was confirmed by observing the collector/drain waveforms in Figure 3 during steady-state operation.

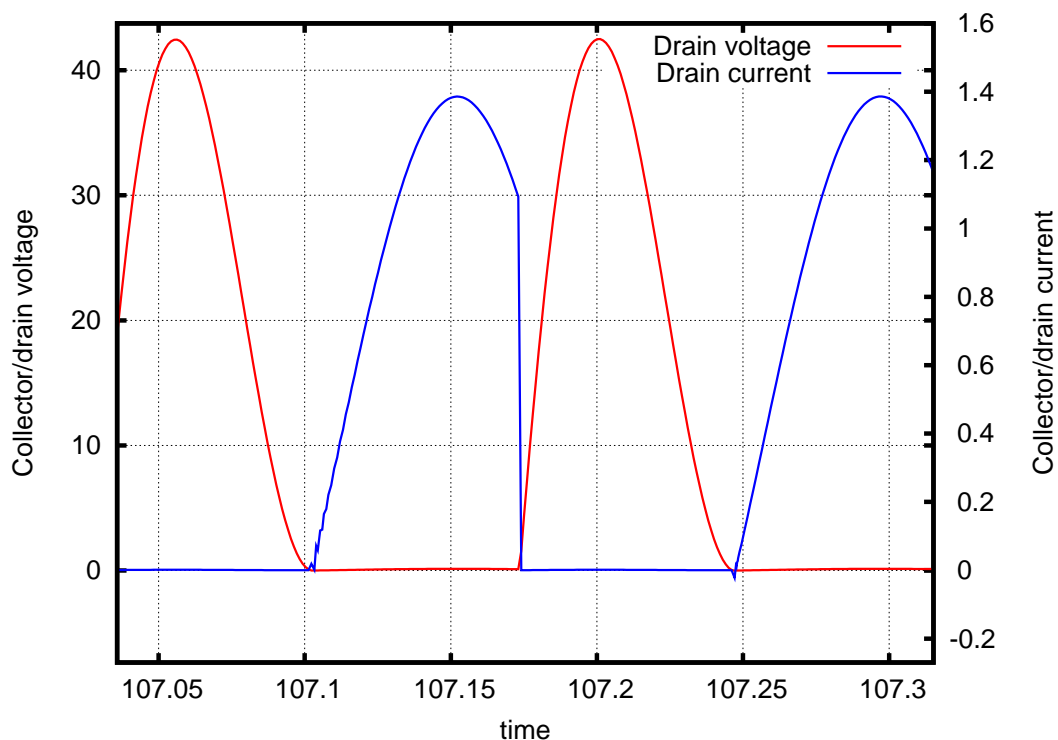


Figure 3: The drain voltage and current waveforms from a SPICE simulation that confirm class-E operation for the component values chosen in the previous section.

Examination of the voltage curve in Figure 3 shows the characteristic “tailing-off” of the voltage just before the switch turns on. Notice also the discontinuity in the drain current as the switch opens up. In this ideal case, there is no overlap between the current and voltage curves, but in real switching devices, there will always be some overlap at this point that reduces the efficiency.

One of the disadvantages of Class-E operation is the high voltage peak that the switch is subjected to. The graph indicates a switch voltage about 3.5 times the value of V_{cc} . This is typical because it is this “voltage kick” that transfers energy to the load. This means, however, that the transistor breakdown voltage must be sized accordingly.

Likewise, the transistor needs to be able to accommodate the current which

rises to a peak value around three times the average DC supply current. The the shunt capacitor must also have low losses and be able to handle significant RF currents. (This is seldom a problem when transistor parasitic capacitance makes up a large part of the required shunt capacitance.)

Finally, we can see how the series resonant circuit and L-match “cleans up” the output sinusoid. Figure 4 shows the current through the load resistor (in blue) as well as the highly irregular switch current (in red).

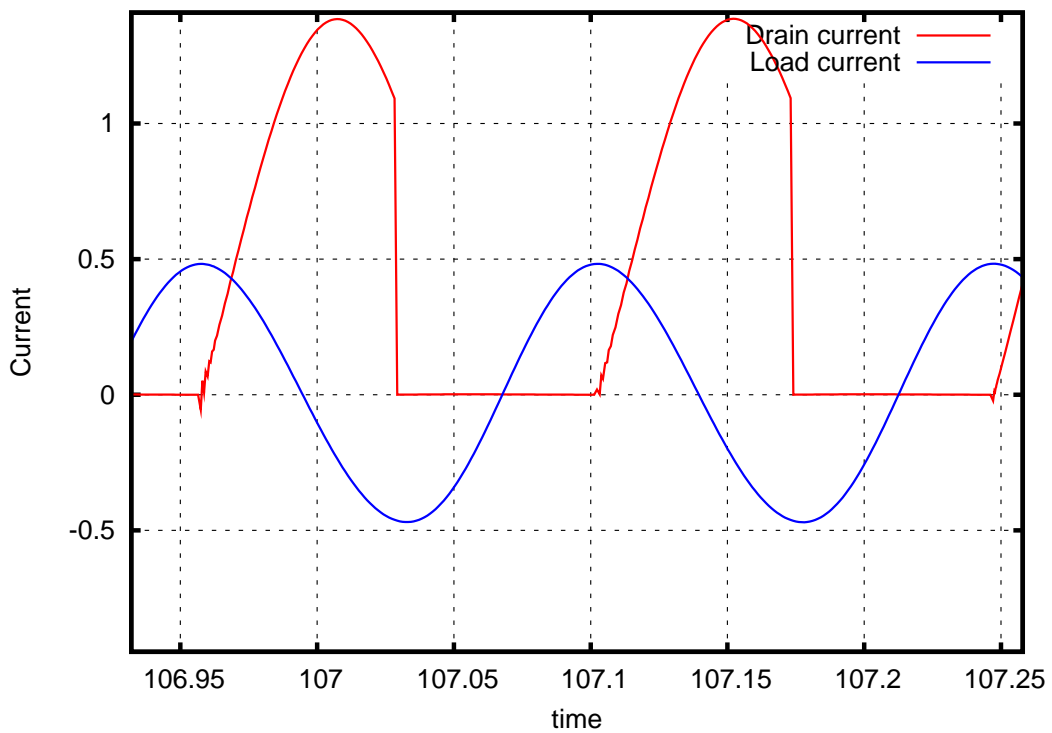


Figure 4: The switch and load current calculated using a SPICE simulation for the component values chosen in the previous section.

References

- [1] A. Grebennikov and A. O. Sokal, *Switchmode RF Power Amplifiers*, Newnes/Elsevier, 2007.